Antrag für ein Graduiertenkolleg

Curvature, Cycles, and Cohomology

Geometric methods in analysis and arithmetic

Universität Regensburg

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1 General Information

1.1 Title

Curvature, Cycles, and Cohomology – Geometric methods in analysis and arithmetic

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1.4 Summary

1.4.1 English

Geometric methods and geometric language provide a common ground of current mathematical research in arithmetic and analysis. On the one hand a lot of progress has been made by transferring ideas between arithmetic and global analysis, and on the other hand global analysis is deeply connected with geometric partial differential equations.

More specifically to the research in Regensburg, concepts as Chern characters or regulators from K-theory to various cohomology groups have enriched our understanding of central results as the Riemann-Roch theorem or the index theorem. Our view on zeta functions occurring in arithmetic and in dynamical systems has profited from structural coincidences in different areas. A characteristic feature of current research in arithmetic and global analysis is the use of the language and methods of homotopy theory, which is employed in arithmetic to study algebraic cycles. Typical aspects of homotopy theory in modern global analysis are twisted K-theory and orientation theory.
On the other hand, the basic figures of global analysis are geometric partial differential equations, most notably the Dirac operator and curvature driven flow equations (Ricci flow, Einstein equations, Willmore flow). The differential geometric and analytic aspects of the description of solutions at singularities or at infinity, which can even be used to prove fundamental theorems in topology (Poincaré conjecture), reflects the common interest of the research groups in Regensburg.
The mathematics department in Regensburg offers a unique research environment for a Graduiertenkolleg centered around the above aspects of global analysis and the concepts of *Curvature, Cycles and Cohomology.* The department consists of three well established research groups in arithmetic geometry, global analysis and partial differential equations. The intended Graduiertenkolleg can build on a great experience in guiding PhD students to the frontiers of research. It will complement the newly founded *Johannes-Kepler-Forschungszentrum für Mathematik* on the educational side. The focus and additional benefit of the Graduiertenkolleg will be the development and teaching of common geometric, analytic, and topological aspects of the interacting fields, the transport of ideas, principles and methods, and their application to central problems in the participating fields themselves. By its joint education programme the Graduiertenkolleg will enable its PhD students in addition to their education in their special field to progress on the basis of a broad background knowledge of different - but from a broader perspective deeply related - fields. As an important aspect it will encourage them to initiate joint projects and interactions. The guest program of the intended Graduiertenkolleg and the support for exchange and collaboration will expose the students to the international research community. It will give them the opportunity to get into contact with leading experts and also to present their own results.

1.4.2 German

Aktuelle Entwicklungen in der Arithmetik und der Analysis sind von einem zunehmenden Gebrauch geometrischer Methoden und Sprache geprägt. Davon zeugen Fortschritte durch den Transfer von Ideen zwischen Arithmetik und Analysis einerseits, und zwischen globaler Analysis und der Theorie geometrischer partieller Differentialgleichungen andererseits.
Als ein typisches Beispiel für die Forschung in Regensburg haben Konzepte wie Chern Charaktere oder Regulatoren, welche K-Theorie mit verschiedenen Kohomologien verbinden, unser Verständnis von grundlegenden Strukturen wie die des Riemann-Roch Theorems oder von Indexsätzen, bereichert. Unsere Sicht auf Zetafunktionen in Arithmetik oder von dynamischen Systemen hat von den strukturellen Gemeinsamkeiten der Theorie in den verschiedenen Gebieten profitiert. Als ein besonders aktuelles Element ist das Eindringen homotopietheoretischer Aspekte in die Arithmetik und die globale Analysis zu erwähnen. Während Homotopietheorie in der Arithmetik besonders zum Studium von Zyklen benutzt wird, liefert sie für die globale moderne Analysis grundlegende Konzepte wie getwistete

K-Theorie oder Orientierungen.
In der anderen Richtung sind die grundlegenden Objekte der globalen Analysis geometrische partielle Differentialgleichungen, wobei hier der Diracoperator sowie Krümmungsflußgleichungen (Ricci-Fluß, Einsteingleichung, Willmorefluß) besonders erwähnt seien. Die differentialgeometrischen Aspekte der Singularitäten von Lösungen und deren Verhalten im Unendlichen (welche etwa im Beweis der Poincaré Vermutung eine wesentliche Rolle spielten) sind im Fokus des gemeinsamen Interesses der Forschungsgruppen.
Die mathematische Fakultät in Regensburg bietet eine attraktive Forschungsumgebung für ein Graduiertenkolleg mit dem Schwerpunkt in diesen Aspekten der globalen Analysis und den Konzepten, welche mit den Schlagwörtern *Krümmung, Zyklen und Kohomologie* umrissen werden.
Die mathematische Fakultät in Regensburg besteht aus drei ausgewiesenen Forschungsschwerpunkten in arithmetischer Geometrie, globaler Analysis und partiellen Differentialgleichungen. Das geplante Graduiertenkolleg wird auf eine große Erfahrung in der Heranführung von Doktoranden an die Fronten aktueller mathematischer Forschung aufbauen können. Es wird das neu gegründete *Johannes-Kepler-Forschungszentrum für Mathematik* auf dem Gebiet der Ausbildung ergänzen. Der Schwerpunkt und zusätzliche Nutzen des Graduiertenkollegs wird die Entwicklung und Vermittlung gemeinsamer geometrischer, analytischer und topologischer Aspekte, der Transport von Ideen, Prinzipien und Methoden und ihre Anwendung auf die zentralen Probleme der teilnehmenden Gebiete selbst sein. Durch sein gemeinsames Ausbildungsprogramm wird das Graduiertenkolleg seine Doktoranden befähigen, sich zusätzlich zu ihrer Ausbildung in ihren Spezialgebieten auf der Basis eines umfassenden Hintergrundwissens verschiedener, aber im weiteren Sinne verwandter Gebiete weiterzuentwickeln. Als wichtiger Aspekt wird es sie ermutigen, gemeinsame Projekte und Interaktionen ins Leben zu rufen. Das Gästeprogramm des geplanten Graduiertenkollegs und die Unterstützung von Austausch und Zusammenarbeit wird die Studenten mit der internationalen Forschungsgemeinschaft in Kontakt bringen. Es wird ihnen die Möglichkeit geben, führende Experten kennenzulernen und auch ihre eigenen Ergebnisse vorzustellen.

1.5 Funding period

October 2010 - March 2015

1.6 Number of PhD students and post docs

We apply for the funding of 10 PhD students and 2 Post-Docs.

2 Profile

2.1 The Research programmes

The combination of keywords in the title

Curvature, Cycles, and Cohomology

indicates a collection of classical concepts of geometry and topology which, appropriately interpreted, become central notions of the areas of mathematics participating in the Graduiertenkolleg. Characteristic common features are the use of a geometric language, of

local-global principles, and ideas from homological and homotopical algebra. These supplement the classical backgrounds from number theory and algebraic geometry on the one end and geometry or partial differential equations on the other.
In addition to leading expertise in these fields Regensburg offers an environment where the exchange and interaction between these fields is desired and practiced.
We will outline the general framework of the participating research project by giving the relevant mathematical interpretation of these three key notions. Much more detailed instances of these concepts will be given in the actual description of the projects.

The geometric concept of **curvature** provides driving forces in geometric evolution problems, measures of the singularity of boundaries, or describes the infinitesimal variation of volume and other global geometric quantities considered in the analytically oriented projects. Curvature quantities give the initial local information for the local-to-global transition, and they are the basic ingredients of the geometric side of spectral geometric questions. Quantities derived from curvature provide the link between the geometry and the topology of manifolds or vector bundles. Their zeros mark the starting point for construction of secondary invariants which are relevant in differential geometry and in arithmetic geometry.

The notion of **cycles** encompasses submanifolds in differential geometry or algebraic cycles in algebraic geometry. Cycles appear as representatives of bordism classes in topology and motivic homotopy, and as objects of variational problems in geometric analysis and measure theory. They are often singular and come with geometric or analytic refinements like associated currents. The microlocal analysis of singular cycles is basic both for the intersection theory in Arakelov geometry and for the understanding of minimizers in geometric variation problems or singularities of flows. The global aspects of cycles appear if one looks at their classes in homology or uses them to define motives and motivic cohomology. Thus cycles are a unifying concept for a fruitful interaction of all participating projects either by coincidence of the objects or by a close relation of the problems, language, and methods.

Cohomology provides global invariants and is the target in the local-to-global transitions. It is the basic tool for coarse-graining and for the passage from geometry to algebra. In the research projects it appears in very different flavours ranging from classical or generalized cohomology in algebraic topology, de Rham cohomology in analysis, sheaf cohomology in algebraic geometry up to motivic cohomology in arithmetic, and higher-categorical ramifications. The comparison of these aspects, e.g., the treatment of Chern classes both analytically (curvature forms of metrics) and topologically (cycles) leads to new interesting results (index theorems). Cohomological and homotopical methods are universal tools used in many projects, and often problems boil down to questions on cohomology.

The **main focus** of the Graduiertenkolleg will be the promotion of developments in the intersection of the participating fields. Let us mention as typical examples

- arithmetic, global analytic and topological aspects of regulators
- global differential geometry, topology, and analysis of Dirac operators
- variational properties of curvature functionals, global differential geometry
- ℓ^2-analytic properties of Galois groups

which will be taken up with more details in the description of the research projects. The composition of the research groups in Regensburg provides a unique and experienced basis to facilitate research in these directions.

2.2 Qualification measures

The intended Graduiertenkolleg will add on a solid classical education of the PhD students in the participating research groups. It will put the emphasis in the training of the students in those mathematical concepts which are common features of the proposed research projects. More concretely, we want to achieve a usable understanding of the topics centered around *Curvature, Cycles, and Cohomology* in the sense decribed in subsection 2.1. We want to enable our students to successfully transport ideas, concepts, and methods between the areas. As a particular important aspect we want to promote the interaction and exchange between our PhD students working in different research groups.
The Graduiertenkolleg will add to the education of PhD students in three levels:

Introductory: The basic joint activity of the Graduiertenkolleg is the weekly

C^3-Ringvorlesung

in which members of the department and selected guest lecturers give introductory and motivational lectures about central aspects in the focus of the Graduiertenkolleg.

Detailed learning: The Graduiertenkolleg will initiate and guide the formation of reading and learning groups with the aim of a more detailed joint education of the PhD students. It will furthermore offer special courses on topics in the intersection of the participating areas.

Horizon broadening: It is intended to organize a

Kolloquium of the Graduiertenkolleg

with guest lectures and reports of the students about their own progress.

2.3 Internationality

The benchmark for the research and educational activities of the mathematical department in Regensburg are the frontiers of first-class international mathematical research. The international recognition of the mathematical department is underlined by frequent invitations of the principal researchers to international conferences, but also by the resonance for activities organized in Regensburg itself. In order to promote the national and international visibility the mathematical department in Regensburg has recently founded the *Johannes-Kepler-Forschungszentrum für Mathematik*. By its guest and workshop programmes it contributes to the international flavor. The Graduiertenkolleg will complement this coordinately by its own guest and workshop programme, but also by supporting the exchange and travel of PhD students.
Internationality of the education on the PhD level in Regensburg can build on the variety of international contacts and cooperations of the scientific teachers as well as on a long tradition to send PhD-students to foreign partners and their activities.
Several faculty members have been on the faculties of universities abroad and can actively help in recruiting and mentoring of international students in Regensburg. It is our believe that students pursuing a scientific career should not only grow up in an international environment but also spend time abroad. Within the PhD program this will be achieved by active participation in international conferences and summer schools and by an encouragement to seek a postdoctoral position abroad at the end of the PhD in Regensburg.

2.4 Selection of applicants

The mathematical department in Regensburg is especially focused in the three areas: *arithmetic geometry, global analysis, and partial differential equations.* These fields are throughout represented by active and recognized researchers. Their interests are intimately linked by the elements of the main focus of the Graduiertenkolleg.
Therefore it was our very natural decision to apply for this Graduiertenkolleg as a joint activity of these three groups, and to gather projects which show their strengths as well as the parallel structures and relationships of the corresponding fields.
The selection of applicants was determined by the necessity to provide a broad mathematical education which deepens the interaction between the participating fields. In addition to the first attempt of this proposal N. Naumann, who recently has accepted a call, becomes an applicant in the formal sense. He already contributed to the research projects previously. With his research interests combining arithmetic geometry and topology and an established collaboration with representatives of the two corresponding research areas his complete integration into the application is an important improvement of the coherence and connectivity.
We are very happy that H. Abels accepted a call to Regensburg recently and contributes to research projects and PhD-topics improving to the coherence, in particular between the areas of partial differential equations and global analysis.
All applicants committed themselves to contribute a considerable effort into the buildup of the Graduiertenkolleg.

3 Research programmes

In the following we describe our research projects in more detail. These projects combine the leading expertise of the involved researchers with the possibility of a joint structured education of PhD students. Work on these problems will foster excellent special training as well as an exchange of ideas and methods between the different fields involved in this proposal. It is understood that the proposed research programme will be further developed and adapted to new progress during the funding period of the Graduiertenkolleg.
Some of our research projects involve a close cooperation with Marco Hien. He has actually only temporary positions in Regensburg. However we would like to point out that his participation will continue independently of his future location of scientific employment.

3.1 Secondary invariants in arithmetic, topology and geometry (U. Bunke, G. Kings, K. Künneman, N. Naumann)

3.1.1 Summary

Generally speaking, secondary invariants are defined if a primary invariant vanishes. Maybe the best known example of this phenomenon is the Chern-Simons invariant of flat vector bundles where the primary invariants, the Chern classes, vanish. A topological example is Adams' e-invariant, where the primary invariant is the degree. In spectral geometry the e-invariant can be expressed in terms of η-invariants. A framework for the systematic study of higher order invariants of framed bordism provides the Adams-Novikov spectral sequence. The resulting connection to the arithmetic of formal groups establishes a very deep and by far not fully understood relation to arithmetic geometry.

In arithmetic geometry secondary invariants are usually closely related to regulators or Abel-Jacobi maps and are extension classes of mixed Hodge structures, or étale sheaves. Main invariants of motives can be seen as extensions of Galois modules or Hodge structures, whose general structure are the subject of very difficult and far reaching conjectures. In Arakelov geometry one studies for example extension classes of hermitian vector bundles over arithmetic curves (whose underlying algebraic extension class is necessarily trivial). A new secondary invariant here is the size, which measures the defect of the orthogonal splitting to be algebraic. In the same spirit, height pairings are secondary invariants, which are only defined between homologically trivial cycles.

The principles of the construction of these secondary invariants are highly related. In the case of the Chern-Simons class a homotopy between the considered connection and the trivial one is used. The definition of the e-invariant uses a zero bordism of the framed manifold as a $Spin^c$-manifold. In arithmetic geometry one uses the difference between a splitting of the Hodge filtration and of the underlying rational structure of the Betti-cohomology to define the extension of mixed Hodge structures. In a similar way the size measures the minimal distance between an algebraic and the orthogonal splitting of an extension of hermitian vector bundles over an arithmetic curve.

The perspectives offered by the four projects give different insights to the secondary invariants which can lead to fruitful interactions. The general formalism of the project secondary invariants in topology has as guiding principles ideas from an arithmetic Riemann-Roch theorem in Arakelov geometry. The Adams-Novikov spectral sequence, as a systematic tool for the study of higher order invariants, connects directly to the arithmetic of formal groups. This is complemented by the arithmetic view of secondary invariants, which come up in the study of regulators and by fundamental invariants of hermitian vector bundles and their extensions on arithmetic schemes like the arithmetic Atiyah class or the size. A substantial interaction with the regulator project 3.6 suggests itself and leads to a deeper understanding of the methods and techniques in this area.

3.1.2 Secondary invariants in topology (U. Bunke, N. Naumann)

A natural framework for the study of secondary invariants in topology is given by smooth extensions of cohomology theories (see [48] for an axiomatic description, [51] for a construction of multiplicative smooth cohomology theories for Landweber exact formal group laws and [95] for a general homotopy theoretic construction). Its first aspects have been introduced in [55]. Motivated by applications in mathematical physics [70], it recently became a very active area of foundational research. In these applications cycles for smooth cohomology classes are considered as fields, and the formalism is used to express locality, quantization conditions and anomalies.
In [48] an absolute formula for Adams' e-invariant and its generalization to families has been given in terms of the integration map in smooth K-theory. The e-invariant is the first of a whole series of invariants of framed manifolds related to steps of the Adams filtration of the stable homotopy groups of spheres. The next level is called f-invariant and has been studied using elliptic cohomology theories, e.g. by [123], [96].
One goal of our recent research is to use smooth extensions of elliptic cohomology theories in the case of the f-invariant in the same way as smooth K-theory in the case of the e-invariant. A first step in this direction has been done in [52]. On the analytic side this

may involve secondary spectral invariants of Dirac operators with vertex operator algebra symmetry. In particular aspects of K-homology for these operators have to be developed. A specific problem is to generalize the construction of the Chiral Dolbeault complex [58] from complex to almost complex manifolds.
The paper [52] also presents the first example tertiary index theorem in the mathematical literature. Its basic ideas can be applied in other situations.
Inside the world of smooth extensions of cohomology theories there are a lot of interesting and important foundational open problems. While [95] shows the existence of smooth extensions in general, multiplicative smooth extensions are only known in a few cases. In [50] we provide an axiomatic characterization for smooth extensions of cohomology theories and show their uniqueness in many cases. The methods of this paper can be applied in other situations to show the existence and uniqueness of natural transformations between smooth extensions of different cohomology theories. This has been applied to the construction of Adams operations [45] and Chern classes [46]
Cycle models for smooth extensions of cohomology theories can serve as a starting point for equivariant generalizations. A first step in this direction is [49], where motivated by the need of physicist's construction of orbifold models of branes we construct and apply a smooth extension of K-theory for orbifolds. It is the basis of an ongoing collaboration with mathematical the physicists (R. Szabo, A. Valentiono) to construct a quantum field theory of selfdual Ramond-Ramond fields on orbifolds.
Another aspect of the theory of smooth extensions of cohomology theories is the striking parallelism with its algebraic (and predating) counterpart (Deligne cohomology and its refinements). The arithmetic geometry side has been developed mainly by Gillet and Soule (see e.g. [81]), while the major analytic contributions are due to Bismut. At the moment we see a coincidence of structures, and it would be an interesting project to find more substantial bridges between the two fields.
As an intermediate step could serve the interesting recent development [139] which leaves the arithmetic aspect aside and builds on the complex analytic and complex differential geometric parts. The main ingredient is the Deligne pairing of line bundles which can naturally be interpreted as a push-forward in a holomorphic version of smooth integral cohomology theory. An important feature of the Deligne pairing, which is not yet reflected in the smooth cohomology theory approach, is its functionality.
One direction of future research is the categorification cycle models of smooth extensions (e.g. the cycles become objects, and the equivalence relations are enhanced to morphisms). The most interesting part of the problem is to exhibit the natural constructions like pull-back, push-forward along oriented maps and products as functors. Carried out properly we expect to get ∞-categories, and the higher order morphisms will give rise to higher order invariants in the same way as the one-morphisms are related to the secondary invariants. This higher-order approach (suggested by Laures $< 2 >$-manifold picture) could be crucial for a proper understanding of the f-invariant mentioned above.
A more precise goal is the construction a functor from an ∞-bordism category to an ∞-smooth K-theory category. At the moment we understand the one-categorial version, and for the higher part we must incorporate ideas of [127], [73], [16]
An ambitious already long-running program related to third order invariants is to understand higher torsion invariants [29] and their algebraic counterpart in a structured manner.

3.1.3 Higher order invariants in topology and derived Shimura varieties (N. Naumann)

The Adams-Novikov spectral sequence is the chief tool for the computation of the stable homotopy of the sphere $\pi_*^s(S^0)$ [142],[135].
Via the Pontrjagin-Thom isomorphism $\pi_*^s(S^0) \simeq \Omega_*^{fr}$, this spectral sequence determines a hierarchy of invariants of framed bordism, each being defined whenever its predecessors vanish. Giving a geometric/analytic description of these invariants is a challenging and wide open problem at the intersection of homotopy theory and analytic geometry, cf. 3.1.2 for a more detailed account of the cases $n = 1, 2$ (the e- and f-invariant) which represent the current edge of knowledge. This perspective exhibits the e-invariant, which is a secondary invariant, as just the beginning of a sequence of higher order invariants, but more is true: As a general strategy, one often tries to express the higher order invariants as invariants of a lower order, evaluated at more complicated geometric objects. Thus the entire hierarchy mentioned above enters the picture in an essential way.
In order to obtain a more geometric underpinning of the Adams-Novikov spectral sequence, following Goerss and Hopkins, one tries to lift the structure sheaf of the stack of formal groups to a sheaf of $\mathcal{E}_\infty$-ring spectra.
We explain two instances in which this program was carried through successfully:

i) Very locally, the Lubin-Tate spectra E_n admit essentially unique $\mathcal{E}_\infty$-structures [82],[83],[63] and the local version of the ANSS becomes

$$(*)\ E_2 = H^*(\mathcal{O}_D^*, W(\mathbb{F}_{p^n})[[u_1, \ldots, u_{n-1}]])^{\mathbb{Z}/n\mathbb{Z}} \Rightarrow \pi_* L_{K(n)} S^0,$$

where the continuous group cohomology of $\mathcal{O}_D^*$, which is the automorphism group of a 1-dimensional formal group Γ of height n over $\mathbb{F}_{p^n}$, is computed with respect to its canonical action on the universal deformation space $Spf(W(\mathbb{F}_{p^n}[[u_1, \ldots, u_{n-1}]])$ of Γ (Lubin-Tate), cf. [54]. It seems interesting perspective to use the recent results of [157] to study this very complicated instance of a compact Iwasawa-module. Specifically, one might hope to prove that $E_2 \otimes \mathbb{Q}$ is as predicted by Hopkins' chromatic splitting conjecture.
We mention that computational experience with $(*)$ in the special case $p = 2, n = 4$ has decisively informed the recent spectacular solution of the Kervaire invariant one problem [93].

ii) To obtain liftings on more global objects one can use a recent profound result in derived algebraic geometry due to Lurie. In particular, this allows to construct unitary Shimura stacks $\mathcal{M}$ such that $\mathcal{O}_{\mathcal{M},et}$ lifts to $\mathcal{E}_\infty$-ring spectra [25]. The resulting $\mathcal{E}_\infty$-ring spectra $TAF := \Gamma(\mathcal{M}, \mathcal{O}_{\mathcal{M},et}^{top})$ where dubbed *topological automorphic forms* since they directly generalize topological modular forms previously constructed by Goerss/Hopkins/Miller by different means. In on-going joint work with Behrens and Hopkins [24] we study string orientations

$$MO < 8 > \to TAF.$$

This leads to constructing Eisenstein measures on unitary groups of signature $(1, n)$, thus relating closely with the project detailed in 3.6.2.

3.1.4 Secondary invariants and polylogarithms (G. Kings)

The relation between special values of L-functions and arithmetical invariants of varieties (expressed by extension of Galois modules or Hodge structures) is one of the leading questions in number theory. The Tamagawa number conjecture gives a very precise formula for the special value in terms of cohomological invariants of the variety and regulators from motivic cohomology to the various cohomology theories involved, as Hodge cohomology, étale cohomology and syntomic cohomology. Deeply connected is the p-adic Beilinson conjecture of Perrin-Riou, which involves p-adic L-functions and Iwasawa theory.

The regulators occurring here usually take their values in certain Ext-groups of mixed Hodge-structures, l-adic sheaves or isocrystals and are hence given by secondary invariants. The relation to the special value of an L-function relies on explicit formulas for the regulator. Thus, the interest here is in explicit formulas for the secondary invariants in special situations. In most known cases they are connected with polylogarithms and Eisenstein series (see [27], [113], [99], [100], [114]).

The case of polylogarithms on $\mathbb{G}_m$ and on elliptic curves is very well understood. The case of higher genus curves is in its infancy and deserves further study. First results were obtained by Goncharov [84] and in [115]. The investigation should start with the description of the Hodge and étale realization of the abelian polylog on curves defined in [115] using the method of quasi Hodge sheaves in [27] and of limits of one-motives in [113]. It is interesting to consider the case of modular curves to relate the polylog to special values of L-functions. The first new case would be the L-function of the symmetric square of a modular form, which one needs to write as a triple product. Such a formula has been obtained by Garrett [80]. This would also be the first instance of a Zagier type conjecture for modular forms.

It should also be possible to relate the polylogarithm on the modular curve in a direct way to the elements defined by Beilinson. It is not difficult to see, that this holds in the case of the dilogarithm.

The cyclotomic polylogarithm also occurs in the higher analytic torsion of certain S^1-bundles, which measures the deviation of the Bismut-Lott Riemann-Roch formula [29] for flat bundles to hold on the level of forms. This gives interesting connections to the project about secondary invariants in topology. One would like to have an explicit connection, which describes the higher torsion in the case of these S^1-bundles as the extension of mixed Hodge structure which gives the cyclotomic polylog.

3.1.5 Secondary invariants in Arakelov geometry (K. Künnemann)

Let $X_{\mathbf{Q}}$ be a smooth, projective variety defined over $\mathbf{Q}$. A standard procedure to associate secondary invariants to rational points, algebraic cycles, and vector bundles on $X_{\mathbf{Q}}$ is to extend these to a model X over Spec $\mathbf{Z}$ and to apply methods from Arakelov geometry. It is well-known from number theory that for a full study of the arithmetic of algebraic cycles and vector bundles on the arithmetic scheme X one has to look at the associated complex manifold $X(\mathbf{C})$ as well. Arakelov geometry offers the most conceptual way to achieve this, by taking the full hermitian differential geometry (hermitian metrics, Green currents, curvature forms, holomorphic torsion) of $X(\mathbf{C})$ into account. For example, an Arakelov intersection theory on arithmetic schemes was developed (Gillet, Soulé extended recently by Burgos, Kramer, Kühn), leading to the computation of heights and height pairings on Shimura varieties (Kudla, Bruinier, Burgos, Kühn, Rapoport), and Arakelov theory was applied to diophantine geometry via the method of slopes (Bost).

In [34], [35] a theory of Arakelov extension groups for metrized vector bundles is developed, which has applications to the reduction theory of lattices and yields a new cohomology theory which is closely related to heights and slope invariants of metrized vector bundles. We introduce and investigate in particular new secondary and refined invariants – the *size* – which is an invariant of certain arithmetic extensions whose underlying algebraic extension is trivial and – *the arithmetic Atiyah class* – which defines an obstruction to the algebraicity of the unitary connection on a hermitian vector bundle that is compatible with its holomorphic structure. The theory of arithmetic extensions is only in the beginning and shall be developed further.

Height pairings for homologically trivial algebraic cycles are secondary invariants which are used for example to describe special values of L-functions (BSD-conjecture, Kudla's program). The global height pairing is the sum of local arithmetic intersection numbers at the archimedean and non-archimedean places. The way of treating archimedean places in arithmetic intersection theory has led Bloch, Gillet and Soulé to apply similiar ideas to p-adic places, thus creating a theory of non-archimedan Arakelov geometry. But in contrast to the archimedean case, the non-archimedean situation allows different methods (e.g., using resolution of singularities and intersection theory on limits of semistable models, or 'analysis' on Berkovich spaces which were followed by different authors (Gubler, Chambert-Loire Thuillier, Zhang,...). These different approaches (e.g., in [32], [85], [86], [117], [156], and [165]) shall be compared. Here the case of desingularized n-fold products of semistable curves seems particularly accessible. Observe that Zhang has given in [166, Sect. 3] a combinatorial description of the intersection theory of divisors in the special fibre of a desingularized self-product of a semistable curve.

3.2 Geometric variational problems and fourth order geometric PDEs (H. Abels, B. Ammann, G. Dolzmann, H. Garcke)

3.2.1 Summary

Geometric variational problems involving higher order integrands and geometric partial differential equations of fourth order have attracted a lot of interest recently. A typical example of a variational problem involving second order derivatives is the problem of finding minimizers of the Willmore energy $\int_\Gamma |H|^2$ of a hypersurface Γ with a given genus. Here H is the mean curvature of the surfaces. The Willmore functional originates from studies in conformal geometry, see Willmore [163], but also appears in general relativity as the main term in the Hawking quasilocal mass, in elasticity theory as a limit energy in a thin plate theory, in cell biology as a surface energy for lipid bilayers and in image processing. The Euler-Lagrange equations of the functional $\int_\Gamma |H|^2$ lead to a fourth order elliptic operator. A major difference between second and fourth order PDEs is that no maximum principle is available and hence many methods developed for second order problems cannot be used. A major difficulty in the analysis of the Willmore energy is its conformal invariance which makes it difficult to apply variational calculus, see e.g. [120]. In addition the highly nonlinear structure of the Euler-Lagrange equations makes it difficult to give a suitable weak formulation - for a recent progress we refer to Rivière [143].

Fourth order geometric *parabolic* partial differential equations appear e.g. as the L^2–gradient flow of the Willmore energy (the Willmore flow), see Kuwert and Schätzle [118]. Due to its highly nonlinear structure the question of the maximal existence time and the behaviour of the Willmore flow near the maximal existence time is in general still open although first results appeared recently in the literature, see [163].

Often additional terms have to be incorporated into the Willmore energy. For example, a line energy can be included or interactions of the curvature of the surface with a vector field defined on the surface have to be accounted for. In the first case singularities on the surfaces have to be expected and in the latter case topological restrictions on vector fields have to be considered.
Another fourth order operator arising in conformal geometry is the so called Paneitz operator. The Paneitz operator is a conformally covariant differential operator, and solutions to related non-linear eigenvalue equations lead to constant Q–curvature. The Q–curvature is a fourth order curvature quantity. The question arises whether a surgery theory is available for the Paneitz operator and, as in the case above, singularities have to be studied for fourth order geometric operators.
Another conformally invariant equation which will be studied is the Dirac-harmonic map equation which is the super-geometric analogue of the harmonic map equation.

3.2.2 Curvature energies and related flows (H. Garcke, H. Abels)

Critical points of the Willmore energy in $\mathbb{R}^3$ fulfill the Euler-Lagrange equation

$$\Delta H + 2H(H^2 - K) = 0 \tag{1}$$

where K is the Gaussian curvature of the surface. The equation (1) is a fourth order elliptic partial differential equation. Geometric variational problems involving the curvature of a surface and elliptic PDEs similar to (1) and its parabolic counterparts will be studied in this project.
A generalization of the classical Willmore energy which appears in the modelling of biological cells is

$$\mathcal{E}(\Gamma) = \int_\Gamma (a_1 + a_2(H - \overline{H})^2 + a_3 K)$$

where $\overline{H}$ is a given function, K is the Gauss curvature, a_1 is the surface tension, a_2 is the bending rigidity and a_3 is the stretching rigidity. If a_3 is a constant the term $a_3 \int_\Gamma K$ is also constant within a fixed topological class. Often on a cell different phases appear and in this case $\mathcal{E}$ has to be modified to

$$\mathcal{F}(\Gamma, \Lambda) = \mathcal{E}(\Gamma) + \int_\Lambda a_4$$

where Λ is a curve on the two-dimensional hypersurface Γ separating the two different phases and a_4 is a line tension constant. In addition, $a_1, a_2, a_3, \overline{H}$ attain different values in both phases, i.e. in particular $\int_\Gamma a_3 K$ is not constant any longer within a fixed topological class. Minimizing $\mathcal{F}$ under constraints is a rather difficult task and no general results are known so far. First results appeared in the physics literature for a radial symmetric situation (see [152]) but they had to enforce regularity assumptions across Λ which are not true in general.
In the project we first plan to study energies of the form

$$\mathcal{G}(\Gamma, \Lambda) = \int_\Gamma a_1 + \int_\Lambda a_4$$

for surfaces with constant enclosed volume. In a second step also variational problems for a general energy $\mathcal{F}$ (first for $a_4 = 0$ and later for $a_4 > 0$) will be studied. Here we first plan to study axisymmetric shapes (see e.g. Seifert [152]) where the Euler-Lagrange equations are considerably simpler. In the general case recent results by Rivière [143] revealing a divergence form of the Euler-Lagrange equation will be helpful. Issues in the general case are existence and characterization of global and local minimizers of $\mathcal{F}$ under constraints on area, enclosed volume, integrated mean curvature, etc. The description of surfaces in terms of spinors gives another interesting view on the Willmore functional [5] and constant mean curvature surfaces [4] and it is also planned to study how this view can be used.

A further aspect are evolution problems in the context of curvature energies. One goal is to study the gradient flow equation for $\mathcal{F}$. This is a difficult task as geometrical evolution equations involving different dimensions are coupled in a nontrivial way and in particular singularities close at Λ are expected. Here experiences in the analysis of triple junctions and boundary contact ([77], [79]) will be helpful and a well-posedness result for the gradient flow equation is a possible PhD project. In particular strong well-posedness locally in time and global well-posedness close to equilibria is of interest in cases with and without singularities as e.g. triple junctions. A numerical method for solving the gradient flow to $\mathcal{E}$ has been proposed in [21] using a suitable notion of conformal maps for polyhedral surfaces. It is planned to analyze this scheme in more detail and to generalize it to the case with line tension.

The gradient flow of the area functional with respect to the Sobolev space H^{-1} leads to the fourth order parabolic PDE

$$V = -\Delta H$$

where V is the normal velocity of an evolving hypersurface. It is planned to study situations with geometric singularities, or more precisely, situations in which three or more surfaces meet at triple lines or quadruple junctions. So far only results for curves in the plane are known, see e.g. [77]. In the higher dimensional case not even well-posedness results have been worked out so far. Establishing well-posedness and the study of stability problems for stationary solutions are possible PhD projects.

A further project is to relax the line energy $\int_\Lambda a_4$ by a Ginzburg-Landau energy

$$\int_\Gamma \Big(\frac{a_4 \varepsilon}{2} |\nabla_\Gamma \phi|^2 + \frac{a_4}{\varepsilon} \psi(\phi) \Big) .$$

Here ε is a small parameter and ψ is a double well potential.

3.2.3 Curvature energies and pattern formation (G. Dolzmann, H. Garcke)

The classical approach to the description of the formation of patterns in phase separating systems is based on a diffuse interface approximation via the Cahn-Hilliard equation. In this project we plan to investigate this evolution on a surface Γ where the separation is driven by an interaction of the phase field and the curvature of the surface. A particular case consists of a system with an additional degree of freedom defined on the surface, namely a director field described by a map from the surface into the unit sphere. In this context a typical assumption is that a uniform director field favors a particular shape of the surface, e.g., a hyperbolic saddle.

A related topic is the question of whether there exist a corresponding sharp interface limit which could be related to harmonic maps. A linearized model inspired by applications was discussed in [22, 23] and we plan to explore further connections and extensions in the framework of Ph.D. projects.

3.2.4 Dynamic stability of critical points of curvature energies (H. Abels, H. Garcke)

Whether a critical point of an energy functional can be observed in real situations depends on its dynamic stability. Here dynamic stability means that solutions for the associated evolution equation/gradient flow with initial values close to the critical point stay close by or even converge for large times to the critical point. The geometry of the critical point and the invariance group of the functional might lead to linearizations at the critical point with zero eigenvalues of a certain order, which have to be taken carefully into account. Stability of critical points is often shown with the aid of a center-manifold analysis of the associated dynamical system. These constructions and arguments are usually quite involved and so far it is not clear how these methods can be applied to situations with non-linear boundary conditions, as it appears e.g. for contact angle problems.
Recently, a generalized principle of linearized stability was developed for quasi-linear parabolic equations by Prüss, Simonett, and Zacher [140] to give a direct and alternative proof of stability in situations, where a center-manifold analysis was necessary before. The goal of this part of the project is to develop these techniques further in the context of the curvature energies of interest in the parts 3.2.2 and 3.2.3. Because of the natural geometric invariances of curvature energies, the associated linearization possess non-trival kernels and might have even some degeneracies, which are due to invariance under the group of diffeomorphisms of the surface. Contact angle conditions and additional line energies as in $\mathcal{G}(\Gamma, \Lambda)$, give rise to non-standard situations, which have to be analyzed carefully. Based on the known results from the center-manifold theory and the generalized principle of linearized stability, we want to develop general techniques to treat stability in such geometric situations, which apply to situations described in the previous subsections.

3.2.5 Dirac-harmonic maps (B. Ammann)

Dirac-harmonic maps are stationary points of the functional

$$E(\phi, \psi) := \int_M |d\phi|^2 + \int_M \langle D\psi, \psi \rangle.$$

Here ϕ is a map from a compact Riemann surface to a Riemannian manifold, ψ is a section of the spinor bundle twisted by ϕ^*TM, and D is the Dirac operator. This equation is the supersymmetric analogue of the classical harmonic map equation. The Dirac-harmonic map equation is a conformally invariant equation. Recently large progress was obtained in [56, 125, 167] about regularity statements and about existence results [105].
Existence results are difficult to obtain, as the functional is neither bounded from below nor from above. Typical results will require the existence of special structures. e.g. the existence of twistor spinors.
Our approach is to use index theoretical obstructions for the existence of such solutions. The methods provides successfully new solutions when the dimension of N is odd. These index theoretical methods have the advantage that they also provide solutions for generic metrics on N. It is likely that these solutions are robust under adding new terms, e.g. terms coming from interactions with other fields.

3.2.6 Paneitz operators (B. Ammann)

In conformal geometry another fourth order elliptic operator, the so-called Paneitz operator, attracted a lot of interest. The Paneitz operator was originally introduced in [137] for

4-dimensional manifolds, and it was generalized by Branson [37] to arbitrary dimensions. It is defined as

$$P_g u = (\Delta_g)^2 u - \mathrm{div}_g(A_g\, du) + \frac{n-4}{2} Q_g u$$

where

$$A_g := \frac{(n-2)^2+4}{2(n-1)(n-2)} \mathrm{Scal}_g \cdot g - \frac{4}{n-2} \mathrm{Ric}_g,$$

$$Q_g = \frac{1}{2(n-1)} \Delta_g \mathrm{Scal}_g + \frac{n^3-4n^2+16n-16}{8(n-1)^2(n-2)^2} \mathrm{Scal}_g^2 - \frac{2}{(n-2)^2} |\mathrm{Ric}_g|^2.$$

The important property of this operator is the transformation law for conformal changes from the metric g to the conformal metric $f^2 g$

$$P_{f^2 g} u = f^{-\frac{n+4}{2}} P_g \left(f^{\frac{n-4}{2}} u \right).$$

The Paneitz operator is an interesting member in the family of conformally covariant differential operators. This family also contains the Dirac operator, acting on spinors, and the conformal Laplace operator L_g defined as $L_g u := \Delta_g u + \frac{n-2}{4(n-1)} \mathrm{Scal}_g\, u$.
These conformally covariant operators have associated conformally invariant non-linear eigenvalue equations $D_g \phi = \lambda |\phi|^{2/(n-1)} \phi$, $L_g u = \lambda u^{\frac{n+2}{n-2}}$, $P_g u = \lambda u^{\frac{n+4}{n-4}}$, and the solutions correspond to constant mean curvature surfaces in Euclidean spaces [4], constant scalar curvature metrics [124] and to constant Q-curvature, the fourth order curvature quantity defined above.
For the Dirac operator [7] and the conformal Laplacian [6] it turned out that bordism and surgery theory is an important tool to study properties of these operators for generic and special metrics. For the Dirac operator D_g we obtain e.g. that the kernel of D_g is as small as the lower bound coming from index theory, for L_g we deduced that a modification of the smooth Yamabe invariant is a bordism invariant.
It is thus natural to ask whether a surgery formula is also available for the Paneitz operator. This task is very well suitable as a PhD thesis. The student should start by adapting known techniques for D_g and L_g to the Paneitz operator. Conditions for removing singularities should be discussed. Estimates on fiber bundles with short fibers should be developed. Once these difficult analytical questions are solved, the project may turn to topological questions.

3.3 Analysis of Dirac operators (B. Ammann, U. Bunke, F. Finster)

3.3.1 Summary

Generalized Dirac operators are first order differential operators on Pseudo-Riemannian manifolds which are elliptic in the Riemannian, and hyperbolic in the Lorentzian context. This class of operators is one of the central objects of global analysis, spectral geometry and index theory. It gives a common framework for the analytic study of the classical operators like the Euler operator, the signature operator, the spin Dirac operator and their twisted, higher or equivariant versions. Major motivations for the theory come from topology and differential geometry on the one hand, and from mathematical physics and general relativity on the other. Keywords in these directions are index theory, the positive scalar curvature problem, and the positive mass property. Dirac operator like objects are also central in non-commutative geometry à la Connes.

In this project we want to study problems in global analysis and spectral geometry with a topological flavour, problems of local analysis of the Dirac operator near singularities, and the structure of particular solutions like harmonic spinors.

3.3.2 Index theory and secondary invariants (U. Bunke)

The global information about Dirac operators is usually encoded in spectral invariants, most prominently the index and its secondary counterparts like eta and torsion invariants which have a toplogical significance, [29], [41]. One of the current streams of research aims at the understanding of the structure of these invariants in terms of smooth extensions of generalized cohomology theories (see e.g. [42], [48]). A striking application of this theory is the intrinsic expression of the Adams e-invariant and Chern-Simons invariants in terms of the eta invariant, see [14], [55] for a classical account. At the moment we can extend these kind of results (which involve manifolds with boundary) to the next level involving corners of codimension 2 using Dirac operators related with elliptic genera. These results provide the motivating examples for a further development of smooth algebraic topology. The philosophy is that the appearance of higher-codimensional corners requires a proper account of higher homotopies.
The algebraic topology of the group of diffeomorpisms of manifolds, or more general, of automorphisms of geometric structures on manifolds, governs the classification of families of such objects. Already the most immediate example of the diffeomorphism group of a sphere is a very interesting example. Its homotopy groups are related with algebraic K-theory. Dirac operators reflect this topology via spectral invariants of families which are derived from suitable superconnections, e.g. analytic torsion forms and eta-forms. Interesting information is derived from deformations to singular situations, e.g. in adiabatic limits. Cases which are important to current developments are

1. the adiabatic limit formula of the equivariant η-form.
2. the asymptotic expansion as $p \to \infty$ of the η-form associated to a family of twisted $Spin^c$-Dirac operators with twisting of the form $E \otimes L^p$, where L is a positive line bundle.

Both topics could be the basis of PhD-projects.
The analytic construction of push-forward maps in K-theory is based on families of Dirac operators. Motivated by models for D-branes with B-field background twisted K-theory became an intensively studied subject. Twisted K-theory classes for non-torsion twists are represented by necessarily infinite-dimensional bundles, and the push-forward requires Dirac operators twisted by such objects. The application to physics is actually based on a smooth refinement of twisted K-theory and the push-forward in this context. The project is to extend our approach [48] to smooth K-theory to the twisted case. First steps in this direction would be to carry out the following:

1. Develop a local index theory for families of Dirac operators which represent push-forwards of twisted K-theory classes, in particular find the local index forms the transgressing η-forms.
2. Show an adiabatic limit formula for these η-forms.
3. Construct a model of smooth twisted K-theory along the lines of [48].

These tasks are well-suited as PhD-projects.

Most of the techniques in the study of topological spectral invariants involve degenerations to singular situations (adiabatic limits, formations of infinite cylinders, boundaries and corners). Hereby it is crucial to understand the behaviour of small eigenvalues and its associated eigenvectors, and the local regularity theory, the other two aspects of the project.

3.3.3 Boundary values for Dirac operators at corners (B. Ammann)

We want to study the regularity of solutions of the Dirac equations close to corners and similar singularities. The approach of [42] works in the rectangular case. Recent applications to locality of field theories (the Stolz-Teichner project), McKay correspondence (e.g. work by A. Degeratu), and numerical modelling of technical devices motivate the study of more general geometric situations. The systematic approach based on conformal blow-up and pseudo-differential operator calculi attached to Lie groupoids already led to considerable progress in the case of second order partial differential equations [8], [9], [10], [11]. In the Lie groupoid approach, the corner are blown-up and one obtains an associated problem on a complete manifold, that is recompactified afterwards, such that the problem is treated in polar coordinates around the singularities. The techniques developed in these articles lead to an improvement of finite element methods for the numerical solution of partial differential equations of second order in 3-dimensional polyhedrons [17]. Elliptic boundary conditions and regularity issues for first order operators, in particular for Dirac operators, shall be studied on these spaces. An important application will be the solution of Maxwell's equation on polyhedrons.
In the case of smooth boundary an important first step is to analyze the Cauchy data space. This is the space whose elements are restrictions of harmonic spinors (without any boundary conditions) to the boundary. Suitable boundary conditions are given by specifying a subspace of the Cauchy data space. To determine the Cauchy data space, one usually enlarges the given manifold to a compact manifold without boundary, called the invertible double.
For boundaries with corners we will blow up conformally the corners before the doubling construction can be carried out. The double thus obtained, will no longer be compact, but its behavior at infinity is uniform.
A large part of the project will be to study to which extend the analysis on the compact double in the case with smooth boundary can be carried over to the non-compact double in the case with corners.

3.3.4 Harmonic spinors and Witten spinors (B. Ammann, F. Finster)

In general relativity, boundary value problems for Dirac operators appear naturally in the derivation of the positive mass theorem and of Penrose-type inequalities, see e.g. [164], [138], [87], [141]. More precisely, the main tool is the so-called Witten spinor, a harmonic spinor with prescribed boundary values in the asymptotic ends. It can also be used for analyzing the geometry of asymptotically flat manifolds of small mass [39], [67]. In order to get more detailed information on curvature, one needs a better understanding of the behavior of the norm of the Witten spinor. A first step in this direction is the level set analysis in [64]. Our goal is to derive curvature estimates which quantify that if the total mass is small, the curvature is small in a weighted L^2-sense, with an exceptional set of small surface area. Combining such an analytic result with topological methods, the aim is to show that a sequences of manifolds with masses tending to zero converges to Euclidean space in a Gromov-Hausdorff sense.

Harmonic spinors will also be studied on compact manifolds. We are interested how the topology of the manifold is related to the dimension of the kernel of the Dirac operator, see [7], and more general, the structure of the small spectrum. In continuation of the work of Gromov-Lawson and Stolz we use bordism theory to get new information about the small spectrum.

3.4 L^2-invariants and equivariant global analysis (B. Ammann, U. Bunke, A. Schmidt)

3.4.1 Summary

The common core of the subprojects combined in this project is the study of spectral properties of operators which are equivariant with respect to an infinite group. These are Dirac type operators or the differentials in combinatorially defined complexes. In particular properties of the small spectrum of these operators are studied using the same invariants and related methods. This is more than is a tight coincidence in the case of de Rham complexes.

The global invariants are usually encoded as elements in the K-theory of a C^*-algebra associated to the group, and numerical invariants are derived by applying traces. The main question is to relate these invariants with global properties of the group and the space on which the group acts.

In analysis the transition from the local properties to global invariants is often based on the locality of resolvents. In geometry exhaustions of the spaces by sequences of compact sets with controlled boundary are important. A very important idea for topological considerations is the approximation of the global object by a sequence of nice quotients.

The study of ℓ^2-invariants is by now a well-established and currently very active branch of mathematics. The following subprojects build on the current state of knowledge. In parts they introduce new types of questions, or they develop new relations with other fields.

3.4.2 L^2-invariants for profinite groups (A. Schmidt)

This project studies the case of a space with a free cocompact action of group via an approximation by a sequence of compact quotients. Their projective limit is a compact space on which a profinite completion of the group acts. Very interesting examples of profinite groups are Galois group which often are pro-p. In this case the relation between the ℓ^2-invariants and cohomology of the quotients with $\mathbb{F}_p$-coefficients is of particular interest.

In this project we want to study to what extend the theory of ℓ^2-invariants can be formulated as to work for profinite groups. Many aspects of the ℓ^2-theory should have analogues in the profinite setting. In particular a generalization of the Atiyah conjecture which asserts that the integral group ring of a torsionfree group embeds into a skew field of unbounded operators is of considerable importance.

Let M be a finite CW-complex and let $\tilde{M}$ be its universal cover. Lück's approximation theorem (see [126]) states that:

$$\beta_k^{(2)}(\pi_1(M) \curvearrowright \tilde{M}) = \lim_{n\to\infty} \frac{\dim_{\mathbb{Q}} H_k(\tilde{M}/\Gamma_n, \mathbb{Q})}{[\pi_1(M) : \Gamma_n]},$$

for any sequence of finite index normal subgroups $\Gamma_{n+1} \subset \Gamma_n \subset \pi_1(M)$ which intersects trivially. In particular, the right hand side is independent of the sequence of subgroups.
Fixing a prime number p, A. Thom has defined natural analogues of ℓ^2-Betti numbers for actions of profinite groups on profinite spaces. One can show that for pro-p-groups, the analogue of Lück's approximation theorem holds for $\beta_1^{(2)}$. It is an interesting problem whether this is true for any profinite group and any k.
The question for the asymptotic behaviour of $\dim_{\mathbb{F}_p} H_k(G_n, \mathbb{F}_p)$ in towers of subgroups is vital in the case that G is a Galois group of local fields or of global fields with restricted ramification (or, more generally, the étale fundamental group of an arithmetic scheme). In the local case the asymptotics is well understood. In the global case it is strongly related to the asymptotic behaviour of certain arithmetic invariants (like the Picard group and the number of completely split primes) in towers of number fields, cf. [136]. There are interesting relations to questions of Iwasawa theory, to Vlăduţ-Tsfasman invariants of 'infinite' global fields [160] and to the $K(\pi, 1)$-property for rings of integers [147], [150].

3.4.3 L^2-Kernels (B. Ammann)

This subproject studies the kernel of equivariant *Spin*-Dirac operators. The main question asks whether the vanishing of index theoretic obstructions imply the existence of a metric for which the corresponding kernel is trivial. An interesting aspect is that the triviality of the kernel has several stages ranging from vanishing of the L^2-kernel over bounds on Novikov-Shubin invariants up to a spectral gap. The corresponding study in the compact case revealed an interesting relation of this question with algebraic topology, in particular bordism theory.
Let M be a compact Riemannian spin manifold of dimension $n = 4k$, and let $\widetilde{M} \to M$ a normal covering with Deck group Γ. The Dirac operator $\widetilde{D}$ acting on L^2-sections of the spinor bundle of $\widetilde{M}$ is selfadjoint. As usual in index theory, one splits $\widetilde{D}$ as

$$\widetilde{D} = \begin{pmatrix} 0 & \widetilde{D}^+ \\ \widetilde{D}^- & 0 \end{pmatrix} .$$

The kernel $\ker \widetilde{D}$ and cokernel $\operatorname{coker} \widetilde{D}^+$ have finite L^2-trace and the Γ-equivariant index theorem [13] states $\hat{A}(M) = \operatorname{tr}_\Gamma \ker \widetilde{D}^+ - \operatorname{tr}_\Gamma \operatorname{coker} \widetilde{D}^+$. The left hand side of this index theorem is an integer which is a topological invariant, whereas the summands on the right will in general depend on the choice of a metric. It seems plausible that for generic metrics either $\operatorname{tr}_\Gamma \ker \widetilde{D}^+$ or $\operatorname{tr}_\Gamma \operatorname{coker} \widetilde{D}^+$ vanishes. This problem has been solved in very few special cases. The case $\Gamma = \{e\}$ could be solved in [7] using preliminary work by [20]. These proofs are based on bordism and surgery techniques. It is expected that similar methods will yield a solution for finite groups with periodic cohomology. As soon as $\#\Gamma = \infty$, the operator $\widetilde{D}$ may have essential spectrum, and many techniques that worked in the case $\Gamma = \{e\}$ do no longer apply. Among infinite Γ the case is $\Gamma = \mathbb{Z}$ much simpler than the case of arbitrary infinite Γ. By using Fourier decomposition one can reduce the $\mathbb{Z}$-equivariant case to a twisted version of the $\Gamma = \{e\}$-equivariant problem see [144].

3.4.4 Equivariant and L^2-index theory (U. Bunke)

The goal of the subproject is to study the K-homology class of an equivariant Dirac operator by localization to subspaces. The study reveals a local-to-global principle which can be used, in good cases, to reduce the calculation of ℓ^2-invariants by ordinary index theory on compact manifolds.
An equivariant generalized Dirac operator D on a G-manifold M represents an equivariant K-homology class $[D] \in K^G(M)$ of M. Globally this class can be studied through its image $\mu([D])$ under the assembly map $\mu : K^G(M) \to K(C^*(G))$. On the other hand it contains information about the local structure of the G-manifold e.g. via its decomposition according to the fixed point sets. The interplay between these points of view is of particular interest, and a lot is known in this direction, see [44], [43] and the appendix of [129], and the literature cited therein. For example, L^2-invariants of the G-manifold can globally be expressed in terms of $\mu([D])$, but also in terms of the local contributions, and this equality is the prototype of L^2-index theorems.
An interesting and successful approach is to interpret geometric constructions with Dirac operators like twisting with bundles or localization to submanifolds in KK-theoretic terms. As a model for this direction one could take [40]. The goal of this project is apply this philosophy in order to understand transverse index theorems in KK-theoretic terms. Cases of particular interest are the constructions of [38], [40], [129]. The essential idea here is to perturb the Dirac operator by adding the Clifford multiplication with a vector field coming from the G-action. Since this term does not graded commute with the principal symbol of the operator a K-theoretic interpretation is still unknown. A successful KK-theoretic understanding of these constructions could shed new light e.g. on the nature of the Blattner formulas for the multiplicities of K-types in discrete series representations or lead to an extension of the results about commuting quantization with reduction to case with singular reductions.

3.5 Semi-Riemannian manifolds with uniform geometries at infinity (H. Abels, B. Ammann, F. Finster)

3.5.1 Summary

In this project we develop analytical tools on Lorentzian manifolds whose geometry at infinity is asymptotic to simple model spaces. Such geometries commonly appear in general relativity and string theory. The simplest examples of such spaces are manifolds which are asymptotic to Minkowski space or to de Sitter or Anti-de Sitter spaces which arise as space-times of isolated gravitating systems, with or without cosmological constant. In (super-) string theory other geometries are important; in particular 10-dimensional Lorentz manifolds that are asymptotic to the product of $3+1$-dimensional Minkowski space with a compact Calabi-Yau manifold of real dimension 6. Riemannian manifolds with uniform geometry at infinity are also of interest in this project, as their product with timelike $\mathbb{R}$ yields important Lorentzian examples.
Our approach is to compactify these spaces. The simplest such compactification is conformal compactification, but also more complicated compactification shall be studied.

3.5.2 Conformal compactification

For conformal compactification, one associates to a given semi-riemannian manifold (M, g) a compact manifold $N \supset M$, possibly with boundary, and one writes g as $\Omega^2 h$ where h is a

metric on N and Ω a non-negative real function, vanishing precisely on $N \setminus M$. For instance, asymptotically Euclidean spaces are compactified by one point to a compact manifold (without boundary). Minkowski space is compactified with a point at space-like infinity, one point each at future and past infinity and two null boundaries joining past infinity to space-like infinity and space-like infinity to future infinity. Such compactifications, called Penrose-compactifications, can also be given for the Schwarzschild and Kerr-Newman geometries which are models for black holes.
One of our aims is to analyze the solutions of hyperbolic equations and to develop the scattering theory in the compactified space-times. For conformally invariant equations like the conformal wave equation or the massless Dirac equation, this problem can be analyzed by considering the initial value problem on null surfaces [128].
We would like to also study equations which are not conformally invariant. Then the main difficulty is that the differential operators after conformal compactification are singular on the boundary. Considering a space-like slice, the Penrose compactification reduces to the conformal compactification as used in [68].

3.5.3 More compactifications

In the above examples, space-like infinity was compactified in a point. For many applications, more sophisticated compactifications (or partial compactifications) are needed. For compactifications of Riemannian manifolds strong pseudodifferential calculi have been established by Melrose, Nistor and collaborators (see e.g. [130], [9]). In these compactifications, a larger boundary at infinity is added, which yields more flexibility. For example Melrose's scattering calculus [131] compactifies Euclidian space to a disk.
Compactifications of Lorentzian manifolds are less understood. In the Lorentzian setting, H. Friedrich [71] developed a compactification for solutions of the vacuum Einstein equations, where space-like infinity is blown up to a cylinder. It is now desirable to put these approaches into a common framework. Some steps towards such results were recently studied in [60] and [159, 132, 158].

3.5.4 Methods, approach and strategy

Classical notions, as wavefront sets of distributions and of operators, bicharacteristics and the symplectic structure on the cotangent space shall be generalized to the points at infinity. Lorentzian manifolds which arise as the product of a "nice" compactified Euclidean manifold with $\mathbb{R}$ as time component have been studied in [159]. In this reference the notion of a bicharacteristic was generalized at infinity to a broken bicharacteristic, and several subcases had to be studied in detail: the ellitpic, the hyperbolic and the glancing case. Similar techniques shall be developed on a larger class of compactified Lorentzian manifolds.
Local solutions of geometric pdes at infinity will be constructed, studied and glued together. Different behavior is expected when infinity is timelike, space-like or null. In the Lorentzian setting, this asymptotic behavior at infinity is closely related to questions as long-time dynamics and rates of decay, which are difficult already in the case of linear hyperbolic equations (see for example [66] and [65]). Applications to scattering theory or the linear stability of model spaces in General relativity (like Schwarzschild or Kerr black holes) are possible applications.
It is also of interest to which extent a pseudoddifferential calculus with smooth coefficients and full asymptotic expansions is needed in these situations. For many constructions only

a first order parametrix and a finite expansion for the calculus rules (e.g. for compositions) is sufficient. This has the advantage that only a finite smoothness of the symbols is needed. In the case of boundary value problems in $\mathbb{R}^n$ a corresponding pseudodifferential calculus was developed in Abels [1]. The calculus was applied by Abels and Terasawa [2] to approximate the resolvent of Stokes-like operators, which was the crucial step to obtain a certain functional calculus and optimal regularity results for the associated instationary Stokes system.

3.6 Regulators in arithmetic, analysis and geometry (M. Hien, U. Jannsen, G. Kings, A. Schmidt)

3.6.1 Summary

The common theme of this project is the study of natural maps, called 'regulator maps' here, between different cohomology theories, and the study of cohomology groups via such regulator maps. Often the related theories are of quite different nature, and the construction as well as the information gained is highly non-trivial.

The most classical regulator map arose in number theory and maps the unit group of the integer ring of a number field to a real vector space, and the regulator is its determinant (with respect to a natural integral lattice of the vector space). Later Borel extended this to arbitrary algebraic K-groups of the integer rings. This was vastly generalized by Beilinson who defined motivic cohomology of arbitrary varieties (via their algebraic K-theory) over number fields and regulator maps into their so-called Deligne cohomology, which is defined analytically. The obtained regulators, again obtained as determinants with respect to lattice structures on both sides, play the main role in the Beilinson-Bloch-Kato conjectures on special values of L-functions. Borel used his regulator maps to compute the algebraic K-groups of integer rings with $\mathbb{Q}$-coefficients. Recently there was big advance in computing these K-groups also integrally, with the further aid of regulator maps into étale cohomology. The same technique also gives a lot of information for arbitrary varieties.

In topology and analysis, the most classical 'regulator map' is the comparison isomorphism between singular cohomology and de Rham cohomology. Again its determinant with respect to natural integral structures gives arithmetic information, the so-called periods. A vast generalization of this comparison isomorphism is the Hilbert-Riemann correspondence relating (regular singular) algebraic D-modules and perverse sheaves. In some pioneering work, Bloch and Esnault considered vector bundles on non-compact curves with irregular singularities at infinity, and defined a period pairing which extends Deligne's pairing for regular singularities. Recently this was extended to arbitrary dimensions, by work of Hien and Mochizuki. By passing from the considered rapid decay homology to a dual cohomology, this can also be considered as a regulator map. The theory of the corresponding periods is still mysterious, as well as the expected analogy with ℓ-adic sheaves.

There is a strong relation with project 3.1 on secondary invariants, by the relationship with the Bloch-Kato conjecture, but also by the fact that often secondary invariants come via regulator maps. E.g., the Abel-Jacobi maps on homologically trivial cycles can be best understood via the total cycle map into étale or Deligne cohomology.

3.6.2 p-adic regulators and zeta functions (G. Kings)

This project is strongly connected with the project "Secondary invariants and polylogarithms". Many questions concerning special values of L-functions and arithmetical invariants of varieties lead to a study of p-adic or syntomic regulators [107]. Even if one is primarily interested in the Tamagawa number conjecture about special values of the complex L-functions, the understanding of the syntomic regulator is necessary for results about the L-values in the region of convergence. Moreover, a lot of deep and interesting results like Kato's work on the Birch-Swinnerton-Dyer conjecture [108], were established with the study of p-adic regulators and explicit reciprocity laws. On the other hand p-adic L-functions and the Iwasawa main conjecture are the main tools in proving the Tamagawa number conjecture in the known cases (see for example [99] and [113]). Decisive for this approach is the explicit computation of the syntomic regulator of very specific K-theory elements, which are constructed in all known cases with the help of the elliptic and the cyclotomic polylogarithms. The p-adic L-functions are also an object of interest in itself and Perrin-Riou has formulated a p-adic Beilinson conjecture.

In the paper [100], a p-adic analogue of the Borel regulator was defined via the Lazard isomorphism and related to the Bloch-Kato exponential and the Soulé regulator to étale cohomology. In particular, this result gives a new description of the syntomic regulator in terms of the Lazard isomorphism. This result is the first step of a project to find an approach to the special values of Dedekind zeta functions without using an Iwasawa main conjecture (which is not available for arbitrary number fields). Tamme [154] has shown that this p-adic Borel regulator coincides with Karoubi's regulator for arbitrary p-adic Banach algebras. As Karoubi's regulator has values in negative cyclic homology [106], it is an interesting project to find the exact relation between negative periodic cyclic homology and syntomic cohomology. Karoubi's defines in fact secondary characteristic classes for generalized (simplicial) flat bundles. This definition also works for p-adic Banach algebras. In the paper [153] Soulé showed how Karoubi's construction is related to the regulator defined by Beilinson with values in Deligne cohomology. A similar comparison should be possible with the syntomic regulator

The results in [19] compute the image of the elliptic polylog under the syntomic regulator for the ordinary part of the modular curve in terms of Eisenstein series of negative weight. This opens the way for proving the p-adic Beilinson conjectures for the p-adic L-functions in the case of CM elliptic curves and modular curves and also for Dirichlet characters following the approaches from [62], [26] and [100]. In particular, it should be possible to describe the cup-product of two syntomic Eisenstein classes in terms of Hida's measure for the Rankin convolution of two modular forms [88].

3.6.3 Finiteness results for motivic cohomology (U. Jannsen, A. Schmidt)

Since several years there exists a well-developed cohomology theory for algebraic varieties and schemes, called motivic cohomology, which works almost as well as the usual singular cohomology theory in topology. It is defined by algebraic cycles, and induces an analogue of the Atiyah-Hirzebruch spectral sequence, here converging to *algebraic* K-theory. But while singular (co)homology groups of complex varieties are known to be finitely generated abelian groups (which is constantly used in Hodge theory), this is not true for the corresponding motivic cohomology groups. But it is conjectured and important in applications, that the K-groups and motivic cohomology groups of *arithmetic schemes* (i.e., schemes of finite type over $\mathbb{Z}$) are finitely generated. Results on this are rare and usually related to deep and fundamental theorems: the finiteness of the class number and Dirichlet's unit theorem are the corner stones of algebraic number theory, and the famous Mordell Weil theorem for abelian varieties is the prerequisite for all further studies in the field as for example the conjecture of Birch and Swinnerton-Dyer. Apart from this, there are the following general results.

Borel's analytically defined regulator map together with reduction theory achieve the finite generation of the motivic cohomology groups of rings of integers of algebraic number fields.

In their work on class field theory for higher-dimensional arithmetic schemes, Bloch, Kato and Saito obtained another finiteness result, concerning the Chow group of zero cycles on regular projective arithmetic schemes. By the results in [151], [149] and [148], this extends to quasi-projective schemes. Over local fields, some finiteness results were obtained in a similar vein in [104].

In a third direction, it follows from the Milnor-Bloch-Kato conjecture on the Milnor K-theory of fields – recently proved by results of Rost and Voevodsky – that for smooth projective varieties over finite fields the motivic cohomology with finite coefficients is finite in a wide range of bi-degrees. For the remaining part, which is more related to geometry, some further finiteness results were shown in [101] and [103], assuming various forms of resolution of singularities. By results in [102], this can be extended to motivic cohomology with integral coefficients for certain classes of varieties, including products of curves and abelian varieties.

In all these different approaches, the main tool is a regulator map, which maps motivic cohomology to another cohomology which can be understood better. In Borel's work it is a priori a real vector space, which then was identified with the Deligne cohomology. This was vastly extended in the setting of the Beilinson conjecture or the Bloch-Kato conjecture, but up to now, the injectivity of these regulator maps is a major problem. In class field theory it is a so called reciprocity map from motivic cohomology to the abelianized fundamental group, which however can again be interpreted as a regulator map, from motivic cohomology to étale cohomology. In the last group of results cited, one considers right away the regulator map from motivic to étale cohomology, and shows that it is an isomorphism in the considered cases. Then one uses known finiteness results for étale cohomology.

We plan to extend these results in three directions: to arithmetic schemes which do not contain a base field, i.e., which have mixed characteristic, to schemes which are not proper, and to motivic homotopy groups, which are an algebraic analogue of the homotopy groups studied in topology. In all these directions of investigation one wants to follow the established lines, but there are technical obstacles related to the (various) definition(s) of motivic cohomology in mixed characteristic, resolution of singularities, p-adic Hodge theory, ramification in higher class field theory, and the complicated definition of algebraic homotopy groups.

But there are also some first results showing some ways how to overcome these difficulties. In [59] the needed controlled and embedded resolution of singularities was established for arbitrary excellent schemes of dimension at most 2. This can hence also be applied in mixed characteristic. A long-term project (by the same authors) has been set up to understand the problems of resolution in higher dimensions. This will also help to extend the results from smooth projective varieties to arbitrary ones. The present focus is on dimension 3, because here are recent complete results on the so-called uniformization, a weaker form of resolution. The aim is to extend this to embedded resolution.

For the mixed case, work in progress by Kato and Sato will help to solve the problems in p-adic Hodge theory.

A new approach to class field theory [112], based on ideas of Wiesend [161], [162], seems to be suited for handling class field theory with ramification. Finally, at least for number rings it was shown in [147], [150], that there are many cases where the higher algebraic homotopy groups vanish after p-localization. Handling motivic cohomology with integral coefficients for arithmetic schemes will be much more difficult than over finite fields. Perhaps finite-dimensionality of motives can help here as well, but one will also need input from Arakelov theory, and in particular height pairings for higher-dimensional cycles.

Because of its many facets, the project is suited for various PhD topics:

In [148] the finiteness of the integral degree 0 singular homology groups of arithmetic schemes was proven by showing that some reciprocity map (=regulator) to the tame abelianized fundamental group is an isomorphism, and that the latter group is finite. There are natural refinements $H_0^{sing,\mathfrak{M}}(X,\mathbb{Z})$ of mentioned singular homology groups, depending on a 'modulus' $\mathfrak{M}$. These are quotients of the group of zero-cycles by an equivalence relation given by divisors with certain congruence conditions on curves $C \subset X$, and can be seen as the higher dimensional generalizations of the ray class groups of number rings. To show the finiteness of these larger groups, a reciprocity isomorphism to some abelianized fundamental group with higher modulus condition should be constructed and studied.

There is a well-known reciprocity map in higher class field theory for varieties over local fields, which maps motivic cohomology to the abelianized fundamental group. In [104] it was shown that the cokernel of this map has a combinatorial description depending on the reduction type, and that its kernel is finite for surfaces (For curves, it is in fact injective.) But the possible structure of the kernel is mysterious. Work of K. Sato shows that the kernel can be non-trivial, whereas work of T. Szamuely shows that it is injective for certain (simple) classes of varieties. It is an interesting PhD project to compute the kernel for various other examples, e.g., for products of curves, to get a guess how its structure is related to more geometric invariants of the variety, and to see if the kernel just depends on the reduction of the variety. One PhD student is already working on this in the case of semi-stable reduction, but, in connection with applying resolution of singularities, there is room for another thesis looking at general reduction.

A promising approach could be to relate finiteness over different (finitely generated) fields (this is in part joint work with M. Kerz). An avatar of this is the result of Artin and Tate which relates the conjecture of Birch and Swinnerton-Dyer (for a curve over a global function field) to the finiteness of the Brauer group (of the surface over a finite field which is the model of the curve). There seems to be a close connection with the generalized Hasse principles considered in [101] and [103], and it would be an interesting project to take the very precise conjectures over finite fields (due to Lichtenbaum, Geisser and Kahn) in the case of a threefold, and to transport them to a function field by choosing a fibration and considering the generic fiber.

3.6.4 Periods for irregular $\mathcal{D}$-modules (M. Hien)

Periods for flat algebraic connections on a smooth algebraic variety over the complex numbers arise by means of comparison between algebraically and analytically defined cohomology groups, the de Rham cohomology of the connection. If the variety is projective, such a comparison theorem results from Serre's GAGA principle. In the non projective case, the analogous result was proven by Deligne [61] under the additional condition on the connection to be regular singular at infinity. The resulting period determinant is of particular interest since it is believed to correspond to the epsilon-factor of ℓ-adic sheaves on varieties over finite fields in a mysterious parallelism between ramification of such ℓ-adic sheaves and flat connections over the complex numbers (see e.g. T.Saito and T. Terasoma [146] where the period determinant is studied under this point of view – still in the regular singular case).

By work of Bloch and Esnault [30] and the results achieved in [91], [90], we have a generalization of Deligne's comparison theorem to the case of arbitrary – i.e. possibly irregular singular – flat connections in form of a perfect pairing between algebraic de Rham cohomology and some modified homology theory on the analytic side. Consequently, we can now define the period determinant in general situations.

The aim of the project is to analyze this period determinant using the analogies to the epsilon-factors mentioned above as a guideline. As for the epsilon-factor, there is a deep understanding due to the work of Laumon [122] in which the analysis of the global ℓ-adic Fourier transform and Laumon's local variants, more precisely a stationary phase formula relating the one to the other, play a prominent role. An analogous stationary phase formula in the situation of $\mathcal{D}$-modules over the complex numbers was proven by Bloch, Esnault [31] and García López [74] independently and further elaborated by Sabbah [145]. Additionally, there are unpublished results of Beilinson, Bloch, Deligne and Esnault giving the analogue of Laumon's results for the period determinant in the case of $\mathcal{D}$-modules on curves. In this project, we aim at generalizations in higher dimensions using the constructions in [91], [90] and thereby extending Saito and Terasoma's work [146] to the case of possibly irregular singular connections. Recent results of T. Mochizuki [133] – which already have entered the work [90] – provide a deep insight into the structure of flat meromorphic connections in any dimension and thus give us a powerful tool for this project. These considerations are part of a current collaboration with C. Sabbah.

Additionally, there are important explicit situation which would be interesting to understand, namely the case of (possibly higher-dimensional generalizations of) confluent hypergeometric differential equations and the Kloostermann differential equations (see Katz [110], [109] for the one-dimensional case). The construction of the latter relies on the convolution of $\mathcal{D}$-modules, the ℓ-adic analogue of which was also examined by Laumon in [122] together with a local variant (see also Katz [111]). We want to analyze these constructions for $\mathcal{D}$-modules with regards to explicit formulae for the convolution in the spirit of Sabbah's result for the Fourier transform (and in analogy to corresponding ℓ-adic results in Laumon [122], (2.7)), the period determinant and applications to explicit examples. Possible PhD projects could be initiated aiming to answer questions of this kind.

Performing the construction of period integrals in [91], [90] in a family $f : X \to Y$ of varieties and a globally integrable connection, leads to applications in the analysis of the solutions of the resulting Gauß-Manin system on the parameter variety Y. In [92], the situation given an irregular singular line bundle on X, with Y being the affine line, has been studied and it is proven that all Gauß-Manin solutions admit integral representations.

We expect similar results in more general situations which we also want to analyze within the scope of this project. Several questions arise that could be examined in PhD projects: a first step should be to consider the case where the irregular line bundle as in [92] is twisted with a regular singular connection before applying the direct image functor requiring to keep control of the monodromy. Results of C. Roucairol and C. Sabbah provide a priori information on the local structure, including the monodromy, of the Gauß-Manin system in this case. In more general situations, i.e. given a more complicated meromorphic connection to start with, much less is known and one of the goals of this project in collaboration with possible PhD-students should be to start these investigations using the deep recent results of T. Mochizuki [133] on the local isomorphism classes of formal connections and their consequences to questions on Stokes structures in arbitrary dimensions, as a starting-point.

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4 Concepts for education

4.1 List of PhD projects

Here is a list if PhD topics arising in the research projects described above. It shall be considered as an offer and will be developed further according to the scientific progress in the involved fields. The actual realization of the proposed projects will also depend on the interests and knowledge of the applying students.

1. (3.1.2) There is an axiomatic description of a smooth extension of a cohomology theory. Classify the smooth extensions of a given cohomology theory. Relevant examples are real K-theory, bordism theories and relative bordism theories like the fibre of the unit $S \to MU$.

2. (3.1.2) Secondary invariants detected by the $\mathbb{Q}/\mathbb{Z}$-version of a cohomology theory can often naturally constructed in the framework of smooth cohomology theories which also often give a geometric ansatz for calculations. Understand the problem of lifting natural transformations between cohomology theories to the smooth extension. Interesting examples are products, certain characteristic classes, or the boundary map $\Sigma^{-1}\overline{MU} \to S$, where $\overline{MU}$ is the cofibre of the unit $S \to MU$.

3. (3.1.2) Develop a theory of Umkehr maps in smooth cohomology theory beyond fibre bundles, e.g. for embeddings. The case of fibre bundles is well-understood in many important examples.

4. (3.1.2) The f-invariant of a framed manifold is calculated in terms of η-invariants associated to the addition choice of an almost complex zero bordism. It would be a deep insight, if one could express this invariant intrinsically in terms of the framed manifold.

5. (3.1.2) The f-invariant is expressed as a formal power series whose coefficients are eta invariants of twisted Dirac operators. The fact, that it a is torsion element in its target group, is a complete mystery from this point of view. The problem is to find an analytic explanation of this fact , e.g. by interpreting the series of Dirac operators as a K-homology class with a vertex operator algebra symmetry.

6. (3.1.3) Study localizations of BP_*BP-comodules [97] using the stack of formal groups. This essentially requires to work out local cohomology in the sense of Grothendieck/Hartshorne for the class of algebraic stacks introduced in [135].

7. (3.1.3) Study the orientation $MO < 8 > \to TAF$ in the case of a unitary Shimura stack $\mathcal{M}$ associated with a group of signature $(1, 1)$. The aim is to obtain an index theoretic interpretation analogous to the case of TMF [12]. When coupled with available computations of $H^*(\mathcal{M}, \omega^{\otimes *})$, this should lead to divisibility results for the indices of twisted Dirac-operators, generalizing those of [94].

8. (3.1.4) Describe the Hodge realization of the abelian polylog on curves in terms of Green functions. Here one should give a conceptual explanation of the polylogarithm functions defined by Goncharov [84] in terms of the extension of quasi Hodge sheaves associated to the abelian polylog on the curve defined in [115].

9. (3.1.4) Describe the étale realization of the abelian polylog on curves in terms of one motives. The étale realization of the elliptic polylogarithm is a limit of one motives [113]. A similar description should be obtained for the étale polylog on curves.

10. (3.1.4) Prove the connection to L-functions in the case of the symmetric square of a modular form. The formula of Garrett gives a description of the L-function of the triple product of modular forms in terms of a convolution by an Eisenstein series on Sp_4. One has to connect this to the explicit formulas by Goncharov in [84] for the polylog.

11. (3.1.4) Study the relation of the polylog on a modular curve to the Beilinson elements. The Beilinson elements are obtained by cup-products of the polylog on the universal elliptic curve and are known to relate to the L-values of modular forms. In the case of the dilogarithm the connection to the polylog on curves follows by an easy calculation. The higher logarithms of the modular curve are not so obviously connected to the Beilinson elements.

12. (3.1.5) Develop an Arakelov and hence analytic analogue of the theory of Chow categories introduced by Franke [69], including the functors f_*, f^* and $f^!$, which would then be a 'geometrization' of the Arakelov Chow groups of Gillet-Soulé or Burgos [53] .

13. (3.1.5) Generalize the results of Zhang in [166, Sect. 3] to triple and n-fold products of semistable curves. Can one use harmonic analysis on the reduction complex as in [165] on the reduction graph? Compare the different approaches to non-archimedean Arakelov geometry described in 3.1.5.

14. (3.2.6) The PhD student should extend the techniques of [128] to non-conformally invariant operators. This leads to the study of singular potentials in the Goursat problem for which new analytic tools have to be developed.

15. (3.3.4, 3.2.6) Construct metrics with large or small kernel of conformally invariant operators with the help of surgery methods. Important examples will be twisted Dirac operators (this part is linked to subproject 3.3.4) and the Paneitz operator (this part is linked to subproject 3.2).

16. (3.2.2) In the PhD project it is planned to investigate surfaces evolving by the fourth order flow $V = -\Delta H$ in situations where surfaces meet at so called triple lines. At these triple lines complicated boundary conditions have to hold and no well-posedness results are known so far. These and stability issues will be studies within the proposed PhD thesis.

17. (3.2.2) Issues of this PhD project are: Derivation of first and second order necessary conditions for the Willmore functional with line tension. Construction of special solution for example in a radically symmetric case. Well-posedness and stability results for the corresponding L^2–gradient flow of the energy.

18. (3.2.2) Prove local in time well-posedness for the gradient flow associated to $\mathcal{G}(\Gamma, \Lambda)$ or even $\mathcal{F}(\Gamma, \Lambda)$. To this end, find a suitable parametrization of the surface and lines on the surface. Study the linearized operator and show maximal regularity in suitable function spaces, which enable the construction of a strong solution locally in time.

19. (3.2.2) De Giorgi proposed to approximate the Willmore energy by a Cahn-Hilliard type phase field energy. It is desirable to also have phase field energies approximating the Willmore energy with line tension. In the PhD thesis different possibilities for such regularizations will be studied first on a formal level and later rigorous Γ–convergence results will be addressed.

20. (3.2.3) This project investigates the relation between harmonic maps (minimizers u of the Dirichlet integral subject to the constraint $|u| = 1$ a.e.) and approximations with a penalization of this constraint. The goal is to formulate rigorous connections in the language of Γ convergence, in particular with respect to the corresponding gradient flow.

21. (3.2.3) This model investigates the (nonlinear) membrane energy (Willmore energy) augmented by a term describing the coupling of the curvature tensor and a director field. The goal is to analyze the existence for the static and the dynamic problem corresponding to a gradient flow of the energy functional.

22. (3.2.3) This projects investigates the interaction of an (almost) tangential vector field on a closed surface with the curvature tensor of the surface. The goal is to understand the necessary singularities in various models and the contribution of these to the total energy.

23. (3.2.4) Show stability or instability of particular rotationally symmetric stationary solutions/global minima. In particular the dynamic stability of minimizers of the Helfrich functional (a Willmore functional with constraints) is of interest. Alternatively, find general criteria for stability/instability in this geometric context.

24. (**??**) Use index theoretical methods to prove the existence of Dirac-harmonic maps

25. (3.3.2) Work out the details for the adiabatic limit of the equivariant η-form. This has applications to to equivariant smooth K-theory and index theory on orbifolds and stacks.

26. (3.3.2) The asymptotic expansion as $p \to \infty$ of spectral theoretic quantities associated to a twisted $Spin^c$-Dirac operator with twisting of the form $E \otimes L^p$ (where L is a positive line bundle) have found very interesting applications, e.g. in Kähler geometry (Donaldsons programme for constant scalar curvature metrics in Kähler classes or in symplectic geometry). The proposed topic is to develop the large p-asymptotic of the η-form for a family of such operators.

27. (3.3.3.) This project studies local and non-local elliptic boundary conditions on complete manifolds with non-compact boundary, in particular the regularity of solutions of Dirac equations at corners. Generalize the classical boundary value properties for Dirac operators on manifolds with smooth compact boundary to smooth non-compact, complete boundary. Then, study boundary value problems for Dirac operators and Maxwell equations on polyhedrons.

28. (3.4.2) The ℓ^2-Betti-numbers of Galois groups of local fields and of global fields with restricted ramification are closely connected to the asymptotic behaviour of certain arithmetic invariants (like the Picard group and the number of completely split primes) in field towers. In the project these relation should be investigated and

ℓ^2-Betti numbers should be calculated in the local case and hopefully also in the global case.

29. (3.4.3) Generalize the results of [7] to the Γ-equivariant setting as described in subproject 3.4.3. The first part of this PhD is mainly of analytic nature. After having solved these analytical problems, the PhD thesis may turn to topological applications.

30. (3.4.4) Give a KK-theoretic interpretation of the construction in [38].

31. (3.4.4) This topic concerns aspects of the concept of *quantization commutes with reduction*. The question is to refine this to the level of K-homology classes, where the case of the action of non-compact groups is of particular interest.

32. (3.5) Study the geometry and analysis on Lorentzian manifolds with uniform geometry at infinity. Of particular interest are the volume growth and geodesics in such manifolds. Further questions concern the wave equations. The idea is to apply adapted pseudodifferential calculi and Lie groupoid methods. The results of this project has applications to General relativity. See 3.5.4 for details.

33. (3.5) Generalize one of the pseudodifferential calculi for compactifications of Riemannian manifolds to the case of non-smooth symbols in the spirit of [1]. In particular show results on composition and parametrix constructions assuming only finite smoothness (e.g. Hölder-smoothness) in the space-time variable of the symbols.

34. (3.6.2) Study the relation between the syntomic regulator and Karoubi's regulator into negative cyclic homology and prove an analogue of Soulé's result [153]. The main technical problem here is to define the right analogue of Karoubi's multiplicative K-theory used by Soulé in the p-adic setting.

35. (3.6.2) Prove the p-adic Beilinson conjecture for ordinary modular forms. Following the approach from [26] and the computations in [19] the main difficulties consist in describing the cup-product of two syntomic cohomology classes in terms of the Eisenstein measure and relate this to the measure of the Rankin convolution by Hida.

36. (3.6.3) Relate the integral singular homology groups of arithmetic schemes with modulus condition $H_0^{sing,\mathfrak{M}}(X,\mathbb{Z})$ to an abelianized fundamental group with higher modulus condition $\pi_1^{ab,\mathfrak{M}}(X)$ via an reciprocity homomorphism; and show finiteness.

37. (3.6.3) Study the kernel of the reciprocity map for smooth projective varieties over local fields with bad reduction. Investigate if it just depends on the reduction of the variety, and try to relate its order to the torsion in the so-called Kato complex.

38. (3.6.4) Consider the Gauß-Manin system associated to a flat meromorphic connection on a variety X with respect to a meromorphic function; generalize [92] to the case of a connection of the form $e^g \otimes R$ with R being regular singular. Start the investigation of more complicated connections – guided by T. Mochizuki's classification results – aiming at a description of local (formal) invariants of the Gauß-Manin system in dimension one in terms of the given data.

39. (3.6.4) Examine the convolution of $\mathcal{D}$-modules on $\mathbb{G}_m$ with a view toward the period determinant (in explicit cases, a rank two Kloostermann system, the basic situation has been described in a Diplomarbeit, 2007). In [122] (2.7), Laumon proves fundamental results on the convolution of ℓ-adic sheaves over a finite field and defines a local variant. One should develop similar results for $\mathcal{D}$-modules.

4.2 Study programme

The study programme of the Graduiertenkolleg will incorporate the already existing well-developed structured parts of PhD education (in particular Oberseminars and reading courses), and will add to their value by creating a larger critical mass in advanced courses. But the main purpose and extra benefit of the Graduiertenkolleg is to organize a joint structured programme of education of PhD students which enlarges the horizon of the participants. Its vision is to promote a deeper learning and training in the language, methods and results not only of the fields of specialization, but also in neighbouring areas which are related in a way not so obvious to the students. By its courses and guest programme, and by the support for exchange it contributes to the overview of the PhD students over the central developments in modern mathematics. Finally, it is supposed to enhance the self-organization and independence of the group of students, and a closer contact between them.

In terms of mathematical contents, the Graduiertenkolleg will in particular support the joint education of the PhD students and provide a special training in the common elements: geometric language, local techniques in geometry, analysis and algebraic geometry, principles of local-to-global transition, and cohomological and homotopical methods.
A deep penetration into a special field and problem is considered as a basic prerequisite for a successful work on a thesis. The wish to ensure in addition a broad mathematical education of PhD-students on the other side is one of the main motivations for this application. The main idea of the educational concept for Graduiertenkolleg is that one can understand and follow the current mathematical development on the basis of a good understanding of the central concepts and principles, and that new contributions can be obtained by developing their features inside the special fields.
The Graduiertenkolleg will organize the education of PhD students on three levels.

4.2.1 Introductory courses and motivational lectures

As a *basic activity* of the Graduiertenkolleg we will organize a

C^3-Ringvorlesung

in which the members of the department and chosen guests give overviews and introductory talks about the participating special fields. The aim of these central courses for all PhD students is to explain the basic concepts, tools and ideas of the participating fields, thus making them more accessible for the non-experts, and inviting further study by themselves. Here is a list of tentative titles and topics for the first year.

Speaker	Title
Abels	Abstract Evolution Equations and Dynamical Systems
Ammann	Surfaces, Spinors and Scalar curvature
Bunke	Topological spectral theory
Dolzmann	Gamma convergence and curvature dominated material behavior
Finster	Hyperbolic equations in relativity
Garcke	Geometric evolution problems
Kings	Regulators and zeta functions
Künnemann	Arithmetic surfaces
Jannsen	Tannakian categories and motives
Naumann	Arithmetic aspects of stable homotopy theory
Schmidt	Higher dimensional class field theory

4.2.2 Horizon broadening

This part supplements the colloquium of the department, which has a long and successful tradition, and which is a place where the modern trends and results of mathematics as a whole are presented by invited specialists.

With the Graduiertenkolleg we plan to add a *colloquium of the Graduiertenkolleg* which combines both aspects, but is more focused on an audience formed by PhD students. This GK-colloquium will be organized by the PhD students themselves. Speakers can be the PhD students reporting on their projects but also invited external speakers paid by the Graduiertenkolleg. In the latter case the talks can be accompanied by a “Fragestunde” in which the speaker gives an introduction to the upcoming talk aimed at a not-so experienced audience.

Finally, we support very much the participation of the PhD students in conferences and workshops on topics of their interest. Conversely, we plan to host regular summer schools at Regensburg with a few distinguished speakers which will also be advertised to students from other Universities in Germany and abroad. The goal of these summer schools is to present the most interesting and active developments or techniques in directions related to the research programme.

4.2.3 Detailed learning

This part of the programme will build on the existing active infrastructure of research oriented Oberseminars of the participating groups which already play a basic role in the education of PhD students. Let us in particular mention the seminars organized in the framework of the Forschergruppe *Algebraic Cycles and Zeta Functions*, the Augsburg-Regensburg seminar and the Erlangen-Regensburg analysis seminar which all contribute to the visibility and interdisciplinarity of the PhD-level education in Regensburg.
The Graduiertenkolleg will encourage the students to form selforganized reading and learning groups. The additional benefit of the Graduiertenkolleg is the chance to provide “interdisciplinary” courses for various groups of students. Here the idea is to explain the

links between structures and tools occurring in different fields. Typically, this will be done by two or more speakers which also can be guests. Examples of topics for such courses could be:

Speaker	**Title**
Abels/Ammann	Pseudodifferential Calculi and Boundary Value Problems
Ammann/Bunke	Analytical and topological aspects of index theory
Ammann/Finster	Selected topics in Mathematical Relativity
Bunke/Kings	Regulators and secondary characteristic classes
Bunke/Künemann	Riemann-Roch theorems
Bunke/Kings	Zetafunctions in Arithmetik and Geometry
Schmidt/Thom	Étale homotopy theory
Jannsen/Naumann	K-theory and motivic cohomology
Garcke/Dolzmann	Modern methods in the Calculus of variations
Ammann/Garcke/Dolzmann	Geometric variational problems
Garcke/Dolzmann/Finster	Regularity theory

4.2.4 Table of mathematical teaching activities and more details

The following table presents the planned educational measures organized by the Graduiertenkolleg in a condensed form.

title	**resp. for org.**	**resp. for content**	**frequency**	**obligatory for**
C^3-Ringvorlesung	Sprecher	Professors of the dept.	weekly	all PhD students
Kolloquium	Sprecher	PhD students	weekly	all PhD students
workshops	selected bord	selected bord	term	all PhD students
oberseminars	research groups	research groups	weekly	dep. on special field
guest lectures	inviting party	guest	not fixed	dep. on special field
learning groups	students, advisors	students, advisors	not fixed	interested students

4.3 Additional qualification measures

1. *Teaching experience* is a key qualification for the academic job marked. The Graduiertenkolleg will encourage the PhD students to become tutors in exercise classes or seminars.

2. The University of Regensburg offers the possibility to take *non-mathematical courses* to acquire further key qualifications or broaden the horizon.

3. Foreign students will be supported by the University to take *german language courses*

4. In very exceptional cases, students with a degree of a Fachhochschule or a Bachelor degree (from a system incompatible with the german one) may be admitted to the Graduiertenkolleg for a preparatory year after which the ordinary admission procedure described in 5.5 follows. The decision is taken by the admission board on demand of the prospective first advisor.

4.4 Guest researchers

We plan to invite short term guests to the Graduiertenkolleg on a regular basis. They will usually speak in the Kolloquium of the Graduiertenkolleg, in an affiliated workshop, or in an Oberseminar. These visits allow our students to see new developments and to set up useful contacts. For the purpose of detailed learning we will invite selected guests for medium periods to give block courses on topics related to the focus of the Graduiertenkolleg. We will also invite advanced doctoral or fresh postdoc students with the idea to get into contact with their results and, in return, to give our students the possibility of presenting themselves in other centers of mathematical research.
The following is a list of senior researchers which we plan to invite:

Name	**from**	**Name**	**from**
Gerd Laures	Bochum	Stefan Schwede	Bonn
Mark Behrends	MIT	Bertrand Toen	Toulouse
Antoine Chambert-Loir	Rennes	Michael Struwe	ETH Zürich
Marc Levine	Northeastern, Boston	Camillo De Lellis	ETH Zürich
Jean-Louis Colliot-Thélène	Orsay	Peter Topping	Warwick
Klaus Ecker	FU Berlin	Luigi Ambrosio	Pisa
Reiner Schätzle	Tübingen	Vincent Maillot	Paris
Jean-Benoit Bost	Orsay	Gerhard Huisken	AEI Potsdam
Damian Roessler	Orsay	Victor Nistor	Penn State
Andras Vasy	Stanford	Helmut Friedrich	AEI Potsdam
Lars Andersson	AEI Potsdam	Rod Gover	Auckland
Mattias Dahl	KTH Stockholm	Jean-Louis Tu	Metz
Sergiu Moroianu	Bucarest	Shuji Saito	Tokyo Univ.
Gebhard Böckle	Essen	Christian Hainzl	Univ. of Alabama
Joel Smoller	Univ. of Michigan	Michael Spieß	Bielefeld
Niky Kamran	McGill Univ. Montreal	Yoshikazu Giga	Tokio
Jean-Marc Fontaine	Orsay	Vincent Cossart	Versailles
Thomas Geisser	USC, Los Angeles	Tom Ilmanen	ETH Zürich
Fabien Morel	LM München	Andrea Malchiodi	SISSA Trieste
Henri Gillet	Chicago	Walter Gubler	Dortmund/Berlin
José Ignacio Burgos Gil	Barcelona	Christophe Soulé	Paris
Emmanuel Humbert	Nancy	Dan Freed	Austin

4.5 Other qualification measures

We encourage our students to participate in international conferences and workshops. Already at an early stage we will direct them to appropriate instructional workshops, DMV-seminars and similar events, e.g. organized by other Graduiertenkollegs.
The Graduiertenkolleg will organize a yearly internal workshop where all PhD students report on their field and their own results. These workshops give the opportunity to train the ability for mathematical presentations and participation in constructive mathematical discussions.

5 Organization, mentoring, quality control, equal opportunity issues

5.1 Supervision and mentoring

Every PhD student will be associated with an advisor team consisting of a first and a second advisor. This team will be formed at the moment of admission. Both members of the team must be members of the mathematical department in Regensburg.
The first advisor is responsible for the supervision of technical aspects of the research project. The second advisor should closely follow the progress of the PhD project and provide additional help.
It is expected that the PhD student stays in close contact with the first advisor. He/she should meet the second advisor regularly each semester. Guided by their advisors they will acquire experience with criteria for good scientific practice.
Female fellows of the GK will be encouraged to form a special mentor relationship with female members on the faculty who have first hand experience with issues of special importance to them. The university offers a lot of assistance and the mentors will be a valuable source of experience, advice, and knowledge.

5.2 Admission and executive board

Decisions about admission of PhD students will be taken at meetings of the admission board consisting of the applying professors. For managing the daily work an executive board consisting of the speaker, its deputy and one representative of the group of PhD students is formed. The members of the executive board will be elected inside their corresponding groups. The executive board will decide about finances, and finally approve guest invitations, workshops, etc.

5.3 Yearly reports of the students

The PhD students are required to submit a written report on the progress of their project (1-2 pages) at the completion of each year after admission. On its basis the admission board will decide about the prolongation.

5.4 Self-organization by the students

Our PhD students will be encouraged to develop their own activities like the organization of reading courses and seminars. They will further take an active part in the organization of the Kolloquium of the Graduiertenkolleg and the workshops.

Each student of the Graduiertenkolleg may propose to invite certain guests, speakers for the Kolloquium of the Graduiertenkolleg, and may suggest topics and speakers for workshops and other activities of the Graduiertenkolleg. Each student may suggest specialized literature to acquire for the library. Final decision will be taken by the executive board.

5.5 Candidate profile and advertisement

5.5.1 Candidate profile

The candidate to be admitted to the Graduiertenkolleg should have an excellent diploma, master, or equivalent degree in mathematics, preferable in an area related to the main research areas of the Graduiertenkolleg. Its quality will be measured by its marks, the results of the thesis, but also by the time needed as a measure for the ability for effective work.
We will also encourage applications of interested students with a corresponding degree in physics (preferable) or other natural sciences satisfying the requirements of the regulations (§3 of the Promotionsordnung (Dr.rer. nat.)) of the University of Regensburg.
The Graduiertenkolleg can accept applicants with a degree from a Fachhochschule (on the basis of a Promotionseignungsprüfung complying the legal requirements §4 of the Promotionsordnung (Dr.rer. nat.)).
In very exceptional cases the Graduiertenkolleg can accept students with a Bachelor degree (this may in particular apply to students coming from the american, british or equivalent system). In this case the student will be admitted for one preparatory year after which a Promotionseignungsprüfung follows.
We will require a working knowledge of written and spoken english. The age limit for admission of PhD students will be 28 (with the usual exceptions) years as required by the DFG .

5.5.2 Selection of PhD students

The board of professors applying for this Graduiertenkolleg form the admission board which will decide about admission of PhD students to the Graduiertenkolleg.
Before application the candidate should contact the desired advisor. They together should work out a research proposal and a working schedule. The research proposal and the schedule must be submitted together with the application.
On the basis of the application and the research proposal the board will decide about an invitation for an interview. The goal of this interview is to check the mathematical excellence of the candidate in general. As a further criterion it will be pre-estimated if the preparation and research proposal of the candidate promises a successful completion in the desired time of about two to three years.
If necessary, the candidates will be ranked according to results of the interview, marks of preceding degrees, and letters of recommendation. With special effort we will encourage and guide qualified female students to apply for a stipend at the Graduiertenkolleg. In this respect we can build on very positive experiences of several colleagues.

1. In the framework of the Forschergruppe they organized workshops for young researchers. Special attention was paid to invite female students. In several cases this boosted the corresponding scientific carrier, e.g. led to invitations to Oberwolfach.

2. Many of us have successfully recruited and mentored female PhD students e.g. C. Bertolin (Jannsen), D. Schiefeneder (Finster), C. Klust (Dolzmann), Ch. Wahl (Bunke), S Fregonese, S. Wieland (Garcke), S. Eisenreich, L. Orton (Kings), St. Wolfrath (Schmidt).
3. We could repeatedly convince female post-doc's to become assistants in Regensburg. Currently these are A. Föglein and N. Große, who will both act as mentors for special questions of female PhD students.

The PhD students will be temporarily admitted for one year. Prolongation will be granted by a decision of the admission board on the basis of the yearly report.

5.5.3 Selection of Post-Docs

The admission board will decide about the Post-Doc positions on the basis of applications and interviews. The Post-Doc positions will be advertised internationally, and in addition the applying professors will actively search for candidates who can considerably contribute to the research and educational programme of the Graduiertenkolleg. One of the two Post-Doc positions will be reserved for a female researcher.

5.5.4 Advertisement

The PhD positions will be advertised internationally. Technically, we will use the homepage of the department, suitable servers like the DMV, AMS, mathematical discussion lists. We will also send out a poster.
An important role will be played by activities of the applying professors in order to set up personal contacts with prospective PhD students. In this connection, let us mention the *Nachwuchskonferenzen* of the Forschergruppe and further activities listed in the table in 6.1.

5.6 Other measures of quality control

5.6.1 Criteria of success

The main indication for quality of the programme is the outcome of excellent theses written by our students. The standard will be in particular measured by a publication of the thesis in a high quality refereed journal or monograph. We also consider a subsequent post-doc employment of our students at prominent places as a sign of scientific success. We will collect information about further employment of our students.
Another important aspect of quality control is that our students finish within the desired period of two to three years. The advisory teams and the system of yearly written reports and presentations are introduced in particular to ensure this goal.

5.6.2 Internal workshop

As one of the measures to intensify the exchange and contact amongst the PhD students working in different fields the Graduiertenkolleg will organize a yearly internal workshop. There, all PhD students supported by the Graduiertenkolleg will give presentations of their current work. This workshop will expose the students to the critics of their fellow students. It will also serve as a measure to improve the social cohesion.

5.6.3 Development of the research programme

The main directions of research described in the programme will be kept fix during the first period of 4.5 years of the Graduiertenkolleg. As all applicants are active partners in the international research process they will identify new important developments and adapt the details of the projects continuously. Formally, the changes have to be approved by the admission board as part of the acception of the research proposals of prospective PhD students.

5.7 Equal opportunity

5.7.1 University wide measures

The new equal opportunity concept of the Regensburg Universität (Jan. 2009) provides a university wide infrastructure for improving the equal opportunity situation and the work-life balance according to the research-oriented standards set by the DFG. The Graduiertenkolleg will build on this infrastructure, but must also contribute from its own resources. We apply for 15000 Euro which will be used according to the needs which can be made precise only later on the basis of the concrete situation of its stipendiaries, collegiates, and post-docs. Possible measures provided by the university infrastructure are

1. childcare service during conferences,
2. shuttle service for children to school and kindergarden,
3. program for children during school holidays,
4. supervised homework for school children on campus,
5. information service,
6. support for the solution of routine problems with SHK (Studentische Hilfskräfte),
7. support for establishing home offices,
8. usage of a nursery on campus (Physics department).

Several of the above measures will be provided by the well established *Familien-Service-Stelle of the University Regensburg*.

5.7.2 Special measures by the Graduiertenkolleg

Compared with the their proportion in the group of diploma students women are still underrepresented among the group of PhD students. This is recognized by the applying professors which will actively encourage qualified women to consider an academic carrier and to apply for a PhD position at the Graduiertenkolleg. In order to promote an encouraging example of successful scientific qualification the Graduiertenkolleg will employ at least one female post doc. By our experience the presence of female academic teachers is of particular importance for attracting female students for a scientific carrier. Knowing that the corresponding situation among the permanent mathematical staff in Regensburg is unsatisfactory we will compensate this by putting special efforts in attracting female guest professors and guest lecturers in the framework of the guest program of the Graduiertenkolleg.

On the basis of the stimulation system for scientific proposals of the University of Regensburg we applied for the support of a one semester guest professorship. With this measure to be realized shortly before or parallel to the start of the Graduiertenkolleg we want to present a prominent example of a female mathematical carrier which, we hope, will help to reach female students already at the beginning of the project. More concretely, we think of inviting e.g. Anna Wienhard (Princeton), Sarah Whitehouse (Sheffield), Konstanze Rietsch (London), Nora Ganter (Illinois).

6 Environment

6.1 Conferences and workshops

The mathematical department in Regensburg is a frequent host of mathematical activities ranging from smaller workshops to medium size conferences. The recently founded *Johannes-Kepler-Forschungszentrum für Mathematik* will provide an institutional basis for the organization of future mathematical activities in Regensburg.
We consider the organization of local workshops and conferences as an important aspect of the education of our PhD students. Such local events provide a time- and cost effective way to confront our students with the recent developments in their and neighbouring fields. The Graduiertenkolleg will therefore also contribute to these events through its guest programme funding.
Here is a list of past and planned activities of the applicants:

F. Finster	Workshop on *Geometry and General Relativity*, July 2004
J. Hornbostel, A. Schmidt	Workshop über *Kobordismus* February 16-18, 2005
U. Jannsen	Workshop über *endlich-dimensionale Motive* February 20-24, 2006
J. Hornbostel, A. Schmidt	Workshop *Algebraic cycles, motives and A1-homotopy theory over general bases* February 13-16, 2007
G. Dolzmann	*Quasiconvexity, quasiregularity and rigidity of gradients*, May 23-26, 2007
G. Kings, K. Künnemann	Workshop *Arithmetic Applications of p-adic Analysis and Rigid Spaces* February 18-22, 2008
B. Ammann, F. Finster	Mini-Workshop on *Scalar curvature and semilinear PDEs in geometry and topology*, June 2008
A. Huber-Klawittter, G. Kings	Nachwuchskonferenz *Arithmetische Geometrie* July 21-25, 2008
G. Dolzmann	*8th Gamm Seminar on Microstructures*, January 16-17, 2009
U. Jannsen	*Finiteness Results for Motives and Motivic Cohomology* February 9-14, 2009
B. Ammann, U. Bunke	Spring School: *Index theory, Lie groupoids and boundary value problems*, March 15-21, 2009
H. Garcke	*Optimization with interfaces and free boundaries* March 23-27, 2009
G. Dolzmann, H. Garcke	One Day Workshop *Biological Membranes* October 2, 2009
K. Künnemann	Workshop *Arakelov Theorey and its Arithmetic Applications* 22.2.2010 - 26.2.2010

6.2 Bachelor and Master programme

The transition from the Diploma to the Bachelor/Master system in Regensburg started in the winter semester 2008/2009.

6.3 Support from university

The support of University of Regensburg for the Graduiertenkolleg consists of

1. office spaces (we expect 5 offices, each shared by two students),
2. the Vitus lecture room (offered by the University guest house) in the historic center of Regensburg, an ideal place for small workshops,
3. computer equipment,
4. access to the library,
5. equal opportunity infrastructure (see above)

6.4 Cooperation with the Forschergruppe Algebraic Cycles and Zeta Functions

The Forschergruppe has set up an intensive, internationally recognized research environment in arithmetic geometry. It contributes to the publicity of the location in the mathematical community. Its presence will be useful to attract the attention of potential PhD students in the starting phase of the Graduiertenkolleg.

7 Funding

7.1 Doctoral students

We apply for the funding of 10 PhD students for a total period of 54 month. In detail we apply for scholarships of maximal height. We justify the amount with the considerable level of the cost of living in Regensburg and the wish to be competitive in relation with other job opportunities of our prospective students in the academic market and in commerce.

7.2 Post-Docs

We apply for the funding of two post-doc positions on the basis of TVL E13. By our experience, formidable and sometimes unexpected progress in the directions described in the research programme is often based on the development of new deep techniques or breakthroughs due to young researchers on the post-doc level. A medium term employment in the Graduiertenkolleg of those (internationally selected, freed from the side conditions imposed by the teaching load of our university scientific assistant positions) post-doc researchers gives an excellent and effective way to make such important developments timely available to the PhD students of the Graduiertenkolleg in Regensburg. In this way the funding of the post-doc positions will contribute to the goal of guiding to PhD theses which are up-to date and internationally recognized. We also expect out post-doc researchers to further progress in their own research and thus contributing to the mathematical visibility of Regensburg.

The selection, finally approved by the admission board, of post-doc researchers, will follow the actual developments. The association to the research projects will be flexibly handled. At the moment (in Germany triggered e.g. by the excellence initiative) active and scientifically recognized post-doc researchers have plenty of attractive job possibilities. In order to be able to persist in this situation of competition and to attract the desired researchers for an employment in Regensburg we apply for post-doc positions instead of scholarships.

7.3 Qualification stipends

We apply for one qualification stipend of 800 EUR (and supplements for family and children) (see 5.5.1).

7.4 Research students

We do not apply for the funding of research students.

7.5 Other costs

7.5.1 Guest programme

We apply for a funding of the guest programme by 20000 EUR per year. It is our plan to have at least one guest (e.g. the speaker of the Kolloquium of the Graduiertenkolleg) each week of the semester. With an estimated average cost of the invitation of about 500 EUR per week and 28 weeks this makes 14000 EUR. The remaining 6000 EUR will be used to cover medium term guests (at least 6 weeks each semester), in particular to contribute to the study program of the Graduiertenkolleg (see section 4.4).

7.5.2 Travel expenses of PhD students

As an integral part of the education of the Graduiertenkolleg we plan to send our PhD students to conferences and workshops. Of further importance are visits of our students at research partners at other German and international universities. We apply for 10000 EUR each year to support travel and participation in conferences of our PhD students.

7.5.3 Summer/winter schools

The Graduiertenkolleg will organize one or two schools each year with international speakers. We apply for the funding of three high level speakers (estimated costs of about 2000 EUR per speaker) for each workshop. Furthermore we want to support the participation of further guests. The estimated costs of a workshop is about 9000 EUR. Tentative topics are

1. Secondary spectral invariants
2. Analysis of natural operators in conformal geometry

In order to advertise the Graduiertenkolleg we will organize an initial workshop right at the start.

7.5.4 Travel to interviews

In order to cover the travel of applicants to the interview in Regensburg we apply for funding. We expect a higher demand in the starting period and after three years.

7.5.5 Advertising

In order to print posters announcing the start of the Graduiertenkolleg and workshops we apply for 1000 EUR each year.

7.5.6 Coordination

For the coordination of the Graduiertenkolleg we apply for a secretary position $\frac{1}{4}$ TVL E6 (8400 EUR per year).

7.5.7 Equal opportunity

We apply for 15000 Euro which will used according to the requirements.

7.5.8 Table

	2010(10-12)	2011	2012	2013	2014	2015(1-3)	sum
guests	5000	20000	20000	20000	20000	5000	90000
travel	2500	10000	10000	10000	10000	2500	45000
workshops/schools	5000	9000	9000	9000	9000	5000	46000
interviews	2000	2000	500	500	1000	500	6500
posters	1000	1000	1000	1000	1000	1000	6000
coordination	2100	8400	8400	8400	8400	2100	37800
equal opportunity	3000	3000	3000	3000	3000	3000	15000
sums	17600	50400	48900	48900	49400	16100	246300

8 Declarations

8.1 Connection to the Forschergruppe

In the period of overlap (little more than one year) with the Forschergruppe *Algebraische Zykel und L-Funktionen* we expect fruitful interaction. Due to its activities like the *Nachwuchskonferenzen* the Forschergruppe can contribute to the attraction of prospective PhD students in the arithmetic areas. The Graduiertenkolleg can not be considered as a continuation of the Forschergruppe.

8.2 Submission of the proposal elsewhere

This proposal or a similar has not been submitted elsewhere.

8.3 Notification of the DFG-liaison professor

Prof. Hans Gruber has been informed about this proposal.

9 Obligations

9.1 General obligations

We agree to

1. follow the DFG-guidelines for good scientific practice,
2. to use the granted funds exclusively for the described goals of the Graduiertenkolleg,
3. to report the DFG about the development of the Graduiertenkolleg and submit the data needed for evaluations.

9.2 Research involving human subjects

none

9.3 Research involving embryonic stem cells

none

9.4 Animal experiments

none

9.5 Genetic engineering experiments

none

10 Signatures

Ammann, Bernd Prof. Dr. rer. nat	
Bunke, Ulrich Prof. Dr. rer. nat	
Dolzmann, Georg Prof. Dr. rer. nat	
Finster, Felix Prof. Dr. rer. nat	
Garcke, Harald Prof. Dr. rer. nat	
Jannsen, Uwe Prof. Dr. rer. nat	
Kings, Guido Prof. Dr. rer. nat	
Künnemann, Klaus Prof. Dr. rer. nat	
Naumann, Niko Prof. Dr. rer. nat	
Schmidt, Alexander Prof. Dr. rer. nat	

Signature of the Legal Representative

Place, Date	Prof. Dr. Thomas Strotthotte Rector of the University of Regensburg

A Curricula vitae and publications of the applicants

A.1 Bernd Ammann

A.1.1 Curriculum vitae

24.2.1969	born in Albstadt
1975–88	School in Albstadt. Abitur.
1988–89	Military Service in Roth (close to Nürnberg) and Meßstetten
1988–89	Partial time studies at Fernuniversität Hagen in mathematics
1989–91	Studies in Freiburg, mathematics (Diplom) and physics (Diplom)
1991	Bachelor (Bakkalaureus) in mathematics in Freiburg
1991–92	Studies in Grenoble, France. Maîtrise des mathématiques
1992–94	Studies in Freiburg
1994	Diplom (mathematics)
1995–98	PhD in mathematics in Freiburg
1998–99	Scientific assistant at Freiburg University
1999–2000	Grant of the DFG for a research year at the Graduate School of City University New York
2000–03	Scientific assistant at Hamburg University
2004	Habilitation, Hamburg
2003–04	Research Grant at the MSRI in Berkeley, California, USA
2004	Temporary Professorship (5 months, April to August) at Bonn University
2004–07	Professor at Université Henri Poincaré, Nancy I, France
2007	Research Grant at the MPI for Gravitational Physic, Potsdam, Germany
2007–today	Professor in Regensburg, Germany

A.1.2 Publications

I. Reviewed Publications

[1] B. Ammann, M. Dahl, E. Humbert. Surgery and the spinorial τ-invariant. To appear in *Comm. Part. Diff. Equations*, arXiv:0710.5673.

[2] B. Ammann. The smallest positive Dirac eigenvalue in a spin-conformal class and cmc immersions. *Comm. Anal. Geom.* 17, 429-479 (2009).

[3] B. Ammann, M. Dahl, E. Humbert. Surgery and harmonic spinors. Adv. Math. 220 no. 2, 523-539 (2009).

[4] B. Ammann, J.F. Grosjean, E. Humbert, B. Morel. A spinorial analogue of Aubin's inequality. *Math. Z.* 260 (2008), 127–151.

[5] B. Ammann, C. Sprouse. Manifolds with small Dirac eigenvalues are nilmanifolds. *Anal. Glob. Anal. Geom.* 31 (2007), 409-425.

[6] B. Ammann, E. Humbert, M. Ould Ahmedou. An obstruction for the mean curvature of a conformal immersion $S^n \to \mathbf{R}^{n+1}$. *Proc. AMS.* 135 (2007), 489–493.

[7] B. Ammann, R. Lauter, V. Nistor. Pseudodifferential operators on manifolds with a Lie structure at infinity. *Anal. of Math.* 165 (2007), 717-747.

[8] B. Ammann, A. Ionescu, V. Nistor. Sobolev spaces on Lie manifolds and regularity for polyhedral domains. *Doc. Math.* 11 (2006), 161–206.

[9] B. Ammann, E. Humbert. The second Yamabe invariant. *J. Funct. Ana.* 235 (2006), 377–412.

[10] B. Ammann, E. Humbert. The first conformal Dirac eigenvalue on 2-dimensional tori. *J. Geom. Phys.* 56 (2006), 623–642.

[11] B. Ammann, E. Humbert, B. Morel. Mass endomorphism and spinorial Yamabe type problems on conformally flat manifolds. *Comm. Anal. Geom.* 14 (2006), 163–182.

[12] B. Ammann, E. Humbert. Positive mass theorem for the Yamabe problem on spin manifolds. *GAFA*, 15 (2005), 567–576.

[13] B. Ammann, R. Lauter, V. Nistor. On the geometry of Riemannian manifolds with a Lie structure at infinity. *Int. J. Math. & Math. Sciences*, 2004:4 (2004), 161–193.

[14] B. Ammann, R. Lauter, V. Nistor, A. Vasy. Complex powers and non-compact manifolds. *Comm. Part. Diff. Eq.*, 29 (2004), 671–705.

[15] B. Ammann, E. Humbert, B. Morel. Un problème de type Yamabe sur les variétés spinorielles compactes. *Compt. Rend. Acad. Sc. Math.*, Ser. I 338 (2004), 929–934.

II. Publications submitted for review

[1] B. Ammann, M. Dahl, E. Humbert. Harmonic spinors and local deformations of the metric. Submitted 03/2009, arXiv:0903.4544.

[2] B. Ammann, M. Dahl, E. Humbert. Smooth Yamabe invariant and Surgery. Submitted 04/2008, arXiv:0804.1418.

[3] B. Ammann, P. Jammes. The supremum of first eigenvalues of conformally covariant operators in a conformal class. Submitted 08/2007, arXiv:0708.0529.

III. Nonreviewed publications

[1] B. Ammann. A surgery formula for the smooth Yamabe invariant. Oberwolfach research report about joint work with M. Dahl and E. Humbert. *Math. Forsch. Oberwolfach Report* 41 (2007), 2413–2417.

A.1.3 Ph. D. Students since 2003

Andres Vargas,	Bonn 2007 (Main advisor, coadvisor: W. Ballmann)
Simon Raulot,	Nancy 2006 (Coadvisor, main advisor: E. Humbert)

A.1.4 Third party funding

10 month research grant at the MSRI Berkeley, USA (August 2003 – May 2004)
6 month research grant at the Max-Planck institute for gravitational physics in Potsdam-Golm, Germany (March 2007 – August 2007)
DFG Sachbeihilfe AM 144/2-1, Small eigenvalues of the Dirac operator, Surgeries and Bordism theory (2008-2010)

A.2 Ulrich Bunke

A.2.1 Curriculum vitae

25.12.1963	born in Berlin
1982–89	student of physics at Humboldt-Universität zu Berlin
1989	diploma in physics, advisor Prof. W. Ebeling (statistical physics, stochastic differential equations)
1989–91	PhD student at the Ernst-Moritz-Arndt Universität Greifswald
1991	PhD, advisor J. Eichhorn (Spectral theory of Dirac operators on open manifolds)
1991–92	guest researcher at the Max-Planck Institut Bonn
1992–96	member of the SFB288 "Differentialgeometrie und Quantenphysik"
1995	Habilitation und Venia Legendi for Mathematics at HU Berlin
1996–2007	professor at the Universität Göttingen
since 2007	professor at the Universität Regensburg

A.2.2 Publications

I. Reviewed Publications

[1] U. Bunke, M. Kreck, T. Schick. A geometric description of smooth cohomology accepted for *Annales Mathematiques Blaise Pascal*, arXiv:0903.5290.

[2] U. Bunke, M. Spitzweck, T. Schick. Periodic twisted cohomology and T-duality, accepted for *Asterisque, arXiv:0805.1459.*

[3] U. Bunke, T. Schick. Smooth K-theory. accepted for Asterisque. arXiv:0707.0046 http://front.math.ucdavis.edu/0707.0046.

[4] U. Bunke, T. Schick, I. Schroeder, M. Wiethaup. Landweber exact formal group laws and smooth cohomology theories. *Algebraic and Geometry Topology* 9 (2009) 1751 – 1790.

[5] U. Bunke. Index theory, eta forms, and Deligne cohomology *Mem. Amer. Math. Soc.*, 198 (2009), vi–120.

[6] U. Bunke, M. Olbrich. Scattering theory for geometrically finite groups *In: Geometry, analysis and topology of discrete groups*, Adv. Lect. Math. (ALM) 6 (2008), 40–136.

[7] U. Bunke, T. Schick. Real secondary index theory *Algebr. Geom. Topol.* , 8 (2008), 1093–1139.

[8] U. Bunke, T. Schick, M. Spitzweck. Inertia and delocalized twisted cohomology. *Homology, Homotopy Appl.*, 10 (2008), 129–180.

[9] U. Bunke, T. Schick, M. Spitzweck, A. Thom. Duality for topological abelian group stacks and T-duality. *In: Cortinas, Guillermo (ed.) et al., K-theory and noncommutative geometry. Proceedings of the ICM 2006 satellite conference, Valladolid, Spain, 2006.* Zürich: EMS. Series of Congress Reports (2008), 227-347

[10] U. Bunke. Orbifold index and equivariant K-homology. *Math. Ann.*, 339(1) (2007), 175–194.

[11] U. Bunke, T. Schick, M. Spitzweck. Sheaf theory for stacks in manifolds and twisted cohomology for S^1-gerbes. *Alg. and Geom. Top.*, 7 (2007), 1007.

[12] U. Bunke, P. Rumpf, T. Schick. The topology of T-duality for T^n-bundles. *Rev. Math. Phys.*, 18(10) (2006), 1103–1154.

[13] U. Bunke, M. Olbrich. On quantum ergodicity for vector bundles. *Acta Appl. Math.*, 90(1-2) (2006), 19–41.

[14] U. Bunke, T. Schick. T-duality for non-free circle actions. *Analysis, geometry and topology of elliptic operators*, 429–466. World Sci. Publ., Hackensack, NJ (2006).

[15] U. Bunke, T. Schick. On the topology of T-duality. *Rev. Math. Phys.*, 17(1) (2005), 77–112.

[16] U. Bunke, X. Ma. Index and secondary index theory for flat bundles with duality. *Aspects of boundary problems in analysis and geometry*, volume 151 of *Oper. Theory Adv. Appl.*, 265–341. Birkhäuser, Basel (2004).

[17] U. Bunke, J. Park. Determinant bundles, boundaries, and surgery. *J. Geom. Phys.*, 52(1) (2004), 28–43.

[18] U. Bunke, P. Turner, S. Willerton. Gerbes and homotopy quantum field theories. *Algebr. Geom. Topol.*, 4 (2004), 407–437.

II. Publications submitted for review

[1] U. Bunke, T. Schick, Uniqueness of smooth extensions of generalized cohomology theories. Submitted 01/2009, arXiv:0901.4423.

[2] U.Bunke. Adams operations in smooth K-theory. Submitted 04/2009, arXiv:0904.4355.

[3] U.Bunke. Chern classes on differential K-theory. Submitted 07/2009, arXiv:0907.2504.

[4] U. Bunke, N. Naumann. Towards an intrinsically analytic interpretation of the f-invariant. Revised version submitted 09/2009, arXiv:0808.0257.

III. Nonreviewed publications

[1] U. Bunke, T. Schick, Smooth orbifold K-Theory. Preprint 2009, arXiv:0905.4181.

[2] U. Bunke, String structures and trivialisations of a Pfaffian line bundle. Preprint 2009, arXiv:0909.0846.

[3] U. Bunke, R. Waldmüller. Analysis on symmetric and locally symmetric spaces (multiplicities, cohomology and zeta functions). *Algebraic groups*, 51–62. Universitätsverlag Göttingen, Göttingen (2007).

[4] U. Bunke, I. Schröder. Twisted K-theory and TQFT. *Mathematisches Institut, Georg-August-Universität Göttingen: Seminars Winter Term 2004/2005*, 33–80. Universitätsdrucke Göttingen, Göttingen (2005).

A.2.3 PhD Students since 2003

Charlotte Wahl	A noncommutative index theorem for a manifold with boundary and cylindrical ends
Michael Schulze	On the resolvent of the Laplacian on functions for degenerating surfaces of finite geometry
Robert Waldmüller	Products and Push-forwards in Parametrized Cohomology Theories
Ansgar Schneider	The local structure of T-duality triples
Ingo Schröder	on-going

A.2.4 Third party funding

Graduiertenkolleg	Gruppen und Geometrie (Göttingen)
DFG BU 1166/5-1	Schwerpunktprogramm Globale Differentialgeometrie "Geometrische und getwistete algebraische Topologie"
DFG BU 1166/7-1	Schwerpunktprogramm Globale Differentialgeometrie "Smooth extensions of generalized cohomology theories"

A.3 Georg Dolzmann

A.3.1 Curriculum vitae

11.2.1964	born in Bonn
1992	Ph.D., Universität Bonn
1994–95	Postdoc at Universität Freiburg
1995–96	Postdoc at Carnegie Mellon University, Pittsburgh
1997–2001	Postdoc at Max Planck Institute for Mathematics in the Sciences, Leipzig
1998–99	Postdoc at California Institute of Technology, Pasadena
2001	Habilitation at Universität Leipzig
2001–06	Professor at the University of Maryland at College Park
since 2006	Professor of Mathematics, Universität Regensburg

A.3.2 Publications

I. Reviewed Publications

[1] S. Bartels, G. Dolzmann, R. H. Nochetto. A finite element scheme for the evolution of orientational order in fluid membranes. To be published in *M^2AN Math. Model. Numer. Anal.* (2009). http://sfb611.iam.uni-bonn.de/uploads/437-komplett.pdf

[2] N. Albin, S. Conti, G. Dolzmann. Infinite-order laminates in a model in crystal plasticity. *Proc. Roy. Soc. Edinburgh A* 139 (2009), 685–708.

[3] S. Conti, G. Dolzmann, C. Klust. Relaxation of a class of variational models in crystal plasticity. *Proc. Royal Soc. London* 465 (2009), 1735–1742.

[4] S. Conti, G. Dolzmann. Gamma-convergence for incompressible elastic plates. *Calculus of Variations and Parital Differential Equations* 34 (2009), 531–551.

[5] J. Adams, S. Conti, A. Desimone, G. Dolzmann. Relaxation of some transversally isotropic energies and applications to smectic A elastomers. *Math. Models Methods Appl. Sci.* 18 (2008), 1–20.

[6] S. Conti, G. Dolzmann, B. Kirchheim. Existence of Lipschitz minimizers for the three-well problem in solid-solid phase transitions. *Ann. Inst. H. Poincaré Anal. Non Linéaire* 24 (2007), 953–962.

[7] S. Conti, G. Dolzmann. Derivation of a plate theory for incompressible materials. *C. R. Math. Acad. Sci. Paris*, 344 (2007), 541–544.

[8] S. Conti, G. Dolzmann, B. Kirchheim, S. Müller. Sufficient conditions for the validity of the Cauchy Born rule close to SO(n). *J. Eur. Math. Soc. (JEMS)*, 8 (2006), 515–539.

[9] G. Dolzmann, J. Kristensen. Higher integrability of minimizing Young measures. *Calc. Var. and Partial Differential Equations*, 22 (2005), 283–301.

[10] C. Carstensen, G. Dolzmann. An a priori error estimate for finite element discretizations in nonlinear elasticity for polyconvex materials under small loads. *Numer. Math.*, 97 (2004), 67–80.

[11] C. Carstensen, G. Dolzmann. Time-space discretization of the nonlinear hyperbolic system $u_{tt} = \operatorname{div}\big(\sigma(Du) + Du_t\big)$. *SIAM J. Numer. Anal.*, 42 (2004), 75–89.

[12] S. Bartles, C. Carstensen, G. Dolzmann. Inhomogeneous Dirichlet conditions in a priori and a posteriori finite element error analysis. *Numer. Math.*, 99 (2004), 1–24.

II. Publications submitted for review

[1] S. Conti, G. Dolzman, S. Müller. The div–curl lemma for sequences whose divergence and curl are compact in $W^{-1,1}$ Submitted 07/2009 to *C.R. Acad. Sci. Paris, Ser. I* http://sfb611.iam.uni-bonn.de/uploads/451-komplett.pdf

A.3.3 PhD Students since 2003

C. Klust: Regularization and scaling in nonconvex models in plasticity, 2007 (ongoing)

A.3.4 Third party funding

DMS0405853	NSF personal grants (2004-2007)
DMS0104118	NSF personal grants (2001-2004)
Forschergruppe FOR-797	“Microplast” der DFG, 2007-2010

A.4 Felix Finster

A.4.1 Curriculum vitae

6.8.1967	born in Mannheim
1987-92	student at Heidelberg University,
	Diplom in mathematics and physics
1992-95	PhD in mathematics at ETH Zurich
1995-96	Post-doc at ETH Zurich
1996-98	Post-doc at Harvard University
1998-2002	Post-doc at the MPI for Mathematics in the Sciences, Leipzig
2000	Habilitation at the University of Leipzig
since 2002	full professor (C4) at the University of Regensburg

A.4.2 Publications

Ia. Reviewed Publications in scientific journals

[1] F. Finster, S. Hoch. An action principle for the masses of Dirac particles. Accepted for *Adv. Theor. Math. Phys.* (2010). arXiv:0712.0678

[2] F. Finster, M. Reintjes. The Dirac equation and the normalization of its solutions in a closed Friedmann-Robertson-Walker universe. *Class. Quantum Grav.* 26, 105021 (2009). arXiv:0901.0602

[3] F. Finster. Causal variational principles on measure spaces. To appear in *J. Reine Angew. Math.* (2009). arXiv:0811.2666

[4] F. Finster, N. Kamran, J. Smoller and S.-T. Yau. Linear waves in the Kerr geometry: a mathematical voyage to black hole physics. *Bull. Amer. Math. Soc.* 46, 635–659 (2009). arXiv:0801.1423

[5] F. Finster. A level set analysis of the Witten spinor with applications to curvature estimates. *Math. Res. Lett.* 16, 41–55 (2009). arXiv:math/0701864

[6] F. Finster, N. Kamran, J. Smoller and S.-T. Yau. A rigorous treatment of energy extraction from a rotating black hole. *Commun. Math. Phys.* 287, 829–847 (2009). arXiv:gr-qc/0701018

[7] F. Finster. From discrete space-time to Minkowski space: basic mechanisms, methods and perspectives. In *Quantum Field Theory*, 263–281, B. Fauser, J. Tolksdorf and E. Zeidler, Eds., Birkhäuser Verlag (2009). arXiv:0712.0685

[8] F. Finster, J. Smoller. Decay of solutions of the Teukolsky equation for higher spin in the Schwarzschild geometry. *Adv.Theor.Math.Phys.* 13, 71–110 (2009). arXiv:gr-qc/0607046

[9] A. Diethert, F. Finster, D. Schiefeneder. Fermion systems in discrete space-time exemplifying the spontaneous generation of a causal structure. *Int. J. Mod. Phys.* A 23:4579-4620 (2008). arXiv:0710.4420

[10] F. Finster, W. Plaum. A lattice model for the fermionic projector in a static and isotropic space-time. *Math. Nachr.* 281 (2008), 803–816. arXiv:0712.067

[11] F. Finster, J. Smoller. A time independent energy estimate for outgoing scalar waves in the Kerr Geometry. *J. Hyperbolic Differ. Equ.* 5 (2008), 221–255. arXiv:0707.2290

[12] F. Finster. On the regularized fermionic projector of the vacuum. *J. Math. Phys.* 49 (2008) 032304. arXiv:math-ph/0612003

[13] F. Finster, M. Kraus. A Weighted L^2-estimate for the Witten spinor in asymptotically Schwarzschild space-times. *Canadian J. Math.* 59 (2007), 943–965. arXiv:math.DG/0501195

[14] F. Finster. Fermion systems in discrete space-time. *J. Phys.: Conf. Ser.* 67 (2007) 012048. arXiv:hep-th/0601140 (2006)

[15] F. Finster. Fermion systems in discrete space-time –outer symmetries and spontaneous symmetry breaking. *Adv.Theor.Math.Phys.* 11 (2007), 91–146. arXivmath-ph/0601039

[16] F.Finster. The Principle of the fermionic projector: an approach for quantum gravity? *Quantum Gravity.* B. Fauser, J. Tolksdorf and E. Zeidler, Eds, Birkhäuser Verlag (2006). arXiv:gr-qc/0601128

[17] F. Finster. A variational principle in discrete space-time –existence of minimizers. *Calc. Var. PDEs* 29 (2007), 431–453. arXiv:math-ph/0503069

[18] F. Finster, H. Schmid. Spectral estimates and non-selfadjoint perturbations of spheroidal wave operators. *J. Reine Angew. Math.* 601 (2006), 71–107. arXiv:math-ph/0405010

[19] F. Finster, N. Kamran, J. Smoller, and S.-T. Yau. Decay of Solutions of the Wave Equation in the Kerr Geometry. *Commun. Math. Phys.* 264 (2006), 465–503. arXiv:gr-qc/0504047

[20] F. Finster, N. Kamran, J. Smoller and S.-T. Yau. An integral spectral representation of the propagator for the wave equation in the Kerr geometry. *Commun. Math. Phys.* 260 (2005), 257–298. arXiv:gr-qc/0310024

[21] F. Finster and M. Kraus. Curvature estimates in asymptotically flat Lorentzian manifolds. *Canadian J. Math.* 57 (2005), 708–723. arXiv:math.DG/0306159

Ib. Monographs

[1] F. Finster. The Principle of the Fermionic Projector. monograph (302 pages), *AMS/IP Studies in Advanced Mathematics* 35 (2006).

II. Publications submitted for review

[1] F. Finster, A. Grotz. The causal perturbation expansion revisited: rescaling the interacting Dirac sea. Submitted 01/2009. arXiv:0901.0334

[2] F. Finster, C. Hainzl. Quantum oscillations prevent the big bang singularity in an Einstein-Dirac cosmology. Submitted 09/2008. arXiv:0809.1693

[3] F. Finster, J. Smoller. Error estimates for approximate solutions of the Riccati equation with real or complex potentials. Submitted 07/2008. arXiv:0807.4406

III. Nonreviewed publications

[1] F. Finster. An action principle for an interacting fermion system and its analysis in the continuum limit. Preprint (2009). arXiv:0908.1542

A.4.3 PhD Students since 2004

D. Batic:	Scattering theory for Dirac particles in the Kerr-Newman geometry, Regensburg 2005
J. Kronthaler:	The scalar wave equation in the Schwarzschild geometry, Regensburg 2007
S. Hoch:	State stability analysis for the fermionic projector in the continuum, Regensburg 2008
present PhD students:	W. Plaum, D. Schiefeneder, A. Grotz

A.4.4 Third party funding

DFG individual program	Linear hyperbolic equations in a black hole geometry (2004–2007)
DFG priority program differential geometry	Curvature problems in Lorentzian manifolds (2005–2009)
DFG individual program	A fermion system in discrete space-time and its continuum limit (2007–)

A.5 Harald Garcke

A.5.1 Curriculum vitae

05.06.1963	born in Bremerhaven
1989	Diploma in mathematics at University of Bonn
1989-1993	Research assistant at SFB 256 "Nichtlineare partielle Differentialgleichungen", University of Bonn
1993	Dr. rer. nat. in mathematics, University of Bonn
1993-1994	Post-doc position at Centre for Mathematical Analysis and Its Applications (University of Sussex, England) with an ESF fellowship
1994-2000	Research assistant at University of Bonn
2000	Habilitation at University of Bonn
2000-2002	Hochschuldozent at University of Bonn
2001	Offers for a professorship in Duisburg and Regensburg
since 2002	Full professor at University of Regensburg

A.5.2 Publications

Ia. Reviewed Publications in scientific journals

[1] J.W. Barrett, H. Garcke, R. Nürnberg. Numerical approximation of gradient flows for closed curves in R^d. Published online in *IMA Journal of Numerical Analysis* (2009), doi:10.1093/imanum/drp005.

[2] H. Garcke, K. Ito, Y. Kohsaka. Stability analysis of phase boundary motion by surface diffusion with triple junction. *Banach Center Publications*, 68, (2009), Polish Academy of Sciences, Warszawa.

[3] M. Alfaro, H. Garcke, D. Hilhorst, H. Matano, R. Schätzle. Motion by anisotropic mean curvature as sharp interface limit of an inhomogeneous and anisotropic Allen-Cahn equation. To appear in *Proc. Royal Society of Edinburgh A* (2009), arXiv:0906.1331.

[4] H. Garcke, K. Ito, Y. Kohsaka. Surface diffusion with triple junctions: A stability criterion for stationary solutions. To appear in *Advances in Differential Equations* (2009). http://www.uni-regensburg.de/Fakultaeten/nat_Fak_I/preprints/Preprints2008/35-2008.pdf

[5] M.H. Farshbaf-Shaker, H. Garcke. Thermodynamically consistent higher order phase field Navier-Stokes models with applications to biological membranes. To appear in *DCDS-S* (2009). http://www.uni-regensburg.de/Fakultaeten/nat_Fak_I/preprints/Preprints2009/12-2009.pdf

[6] J.W. Barrett, H. Garcke, R. Nürnberg. Numerical approximation of anisotropic geometric evolution equations in the plane. *IMA Journal of Numerical Analysis*, 28 (2) (2008), 292–330.

[7] J.W. Barrett, H. Garcke, R. Nürnberg. On sharp interface limits of Allen-Cahn/Cahn-Hilliard variational inequalities. *Discrete and Continuous Dynamical Systems Series S*, 1 (1) (2008), 1–14.

[8] J.W. Barrett, H. Garcke, R. Nürnberg. On the parametric finite element approximation of evolving hypersurfaces in R^3. *J. Comput. Phys.*, 227 (2008), 4281–4307.

[9] J.W. Barrett, H. Garcke, R. Nürnberg. A variational formulation of anisotropic geometric evolution equations in higher dimensions. *Numerische Mathematik*, 109 (2008), 1–44.

[10] H. Garcke, K. Ito, Y. Kohsaka. Nonlinear stability of stationary solutions for surface diffusion with boundary conditions. *SIAM J. Math. Anal.*, 40, no. 2 (2008), 491–515.

[11] H. Garcke. The Γ-limit of the Ginzburg-Landau energy in an elastic medium *AMSA* 18 (2008), 345-379.

[12] H. Garcke, B. Nester, B. Stinner, F. Wendler. Allen-Cahn systems with volume constraints. *Mathematical Models and Methods in Applied Sciences (M3AS)*, 18, Issue: 8 (2008), 1347-1381, DOI: 10.1142/S0218202508003066.

[13] H. Garcke, R. Haas. Modelling of non-isothermal multi-component, multi-phase systems with convection. *STMS Books*, Phase Transformations in Multicomponent Melts. Wiley-VCH Verlag, Weinheim (2008), 325-338.

[14] H. Garcke, B. Nestler, F. Wendler, M. Selzer, B. Stinner. Phase-field model for multiphase systems with preserved volume fractions. *Phys. Rev. E*, 78, 011604 (2008).

[15] J.W. Barrett, H. Garcke, R. Nürnberg. Parametric approximation of Willmore flow and related geometric evolution equations. *SIAM J. Sci. Comp.*, 31, no. 1 (2008), 225-253.

[16] H. Garcke, Y. Kohsaka, D. Sevcovic. Nonlinear stability of stationary solutions for curvature flow with triple junction. Calculus of Variations and Partial Differential Equations. To appear in *Hokkaido Mathematical Journal*, Hokkaido University EPrint Servers # 900 (2008).

[17] J.W. Barrett, H. Garcke, R. Nürnberg. A phase field model for the electromigration of intergranular voids. *Interfaces and Free Boundaries*, 9 (2007), 171–210.

[18] J.W. Barrett, H. Garcke, R. Nürnberg. On the variational approximation of combined second and fourth order geometric evolution equations. *SIAM J. Scientific Comput.*, 29, Issue 3 (2007), 1006–1041.

[19] J.W. Barrett, H. Garcke, R. Nürnberg. A parametric finite element method for fourth order geometric evolution equations. *Journal of Computational Physics*, 222, Issue 1 (2007), 441–467.

[20] H. Garcke, R. Nürnberg, V. Styles. Stress and diffusion induced interface motion: Modelling and numerical simulations. *European Journal of Applied Math.*, 18 (2007), 631–657.

[21] J.W. Barrett, H. Garcke, R. Nürnberg. Finite element approximation of a phase field model for surface diffusion of voids in a stressed solid. *Math. Comp.*, 75 (2006), 7–41.

[22] H. Garcke, S. Wieland. Surfactant spreading on thin viscous films: Nonnegative solutions of a coupled degenerate system. *SIAM J. Math. Analysis*, 37 (6)(2006), 2025–2048.

[23] H. Garcke, B. Stinner. Second order phase field asymptotics for multi-component systems. *Interfaces and Free Boundaries*, 8 (2006), 131–157.

[24] H. Garcke. On a Cahn-Hilliard model for phase separation with elastic misfit. *Ann. de l'IHP - Analyse non lineaire*, 22 (2) (2005), 165–185.

[25] H. Garcke, K. Ito, Y. Kohsaka. Linearized stability analysis of stationary solutions for surface diffusion with boundary conditions. *SIAM J. Math. Anal. 36*, No. 4 (2005), 1031–1056.

[26] H. Garcke. Mechanical effects in the Cahn-Hilliard model: A review on mathematical results. in "Mathematical Methods and Models in phase transitions", ed.: Alain Miranville, *Nova Science Publ.* (2005), 43–77.

[27] H. Garcke, B. Nestler, B. Stinner. Multicomponent Alloy Solidification: Phase-Field Modelling and Simulations. *Phys. Reviews E, Vol. 71*, No. 4 (2005), 041609-1–6.

[28] H. Garcke, U. Weikard. Numerical approximation of the Cahn-Larché equation. *Numer. Math.*, 100, Number 4 (2005), 639–662.

[29] H. Garcke, V. Styles. Bi-directional diffusion induced grain boundary motion with triple junctions. *Interfaces and Free Boundaries*, 6 (3) (2004), 271–294.

[30] H. Garcke, B. Nestler, B. Stinner. A diffuse interface model for alloys with multiple components and phases. *SIAM J. Appl. Math.*, 64 (2004), 775–779.

Ib. Monographs

[1] C. Eck, H. Garcke, P. Knabner. Mathematische Modellierung. *Springer-Lehrbuch*, Springer Verlag, Berlin–Heidelberg 2008.

II. Publications submitted for review

[1] J.W. Barrett, H. Garcke, R. Nürnberg. On stable parametric finite element methods for the Stefan problem and the Mullins-Sekerka problem with applications to dendritic growth. Submitted 10/2009. http://www.uni-regensburg.de/Fakultaeten/nat_Fak_I/preprints/Preprints2009/21-2009.pdf

[2] J.W. Barrett, H. Garcke, R. Nürnberg. Finite element approximation of coupled surface and grain boundary motion with applications to thermal grooving and sintering. Submitted 09/2009. http://www.uni-regensburg.de/Fakultaeten/nat_Fak_I/preprints/Preprints2009/11-2009.pdf

[3] L. Blank, M. Butz, H. Garcke. Solving the Cahn-Hilliard variational inequality with a semi-smooth Newton method. Submitted 05/2009. http://www.uni-regensburg.de/Fakultaeten/nat_Fak_I/preprints/Preprints2009/07-2009.pdf

[4] L. Blank, H. Garcke, L. Sarbu, V. Styles. Primal-dual active set methods for Allen-Cahn variational inequalities with non-local constraints. Submitted 04/2009. http://www.uni-regensburg.de/Fakultaeten/nat_Fak_I/preprints/Preprints2009/06-2009.pdf

[5] J.W. Barrett, H. Garcke, R. Nürnberg. Parametric approximation of surface clusters driven by isotropic and anisotropic surface energies. Submitted 04/2009. http://www.uni-regensburg.de/Fakultaeten/nat_Fak_I/preprints/Preprints2009/04-2009.pdf

A.5.3 PhD Students since 2003

Sandra Wieland:	Modellierung und mathematische Analyse kontaminierter dünner Flüssigkeitsfilme, Bonn 2003
Björn Stinner:	Derivation and analysis of a phase field model for alloy solidification, Regensburg 2005
Tobias Kusche:	Spectral analysis for linearizations of the Allen-Cahn equation around rescaled stationary solutions with triple junctions, Regensburg 2006
Robert Haas:	Modeling and Analysis for general non-isothermal convective Phase Field Systems, Regensburg 2007
David Jung Chul Kwak:	Asymptotischer Limes des Cahn-Hilliard-Modells unter Berücksichtigung elastischer Effekte, Regensburg 2008
On going:	
Martin Butz:	Optimal control of interfaces
Daniel Depner:	Geometric evolution equations with triple junctions
Serena Fregonese:	Geometric evolution equations on submanifolds
André Oppitz:	Mathematische Modellierung und Devicesimulation für Halbleiterbauelemente

A.5.4 Third party funding

DFG BL 433/2-1 DFG GA 695/5-2	Optimization problems governed by Cahn-Hilliard equations, joint project with Luise Blank as part of the Schwerpunktprogramm Optimierung mit partiellen Differentialgleichungen (2006-2012)
DFG GA 695/1	Multiple scales in phase separating systems with elastic misfit as part of the Schwerpunktprogramm Analysis, Modellbildung und Simulation von Mehrskalenproblemen (2000-2007)
DFG GA 695/2	Project Analysis, modelling and simulation of multi-scale, multi-phase solidification in alloy systems as part of the Schwerpunktprogramm Analysis, Modellbildung und Simulation von Mehrskalenproblemen (2001-2007)
DFG GA 695/3	Phasenfeldmodellierung der Erstarrung in mehrkomponentigen und mehrphasigen Legierungssystemen as part of the Schwerpunktprogramm Phasenumwandlungen in mehrkomponentigen Schmelzen (2002-2007)

A.6 Uwe Jannsen

A.6.1 Curriculum vitae

11.03.1954	born in Meddewade, Schleswig-Holstein
09/72–08/80	study of Mathematics and Physics, Universität Hamburg
07/78	Diploma in Mathematics, Hamburg
09/80	PhD in Mathematics, Hamburg
06/88	Habilitation, Regensburg
08/80–09/89	Assistant and Assistant Professor at the University of Regensburg
09/83–08/84	Postdoc Fellow at Harvard University, Cambridge, USA
10/89–01/91	Research Professor at the Max-Planck-Insitut für Mathematik, Bonn
02/91–04/99	Full Professor at the University of Köln
since 04/99	Full Professor at the University of Regensburg

A.6.2 Publications

I. Reviewed Publications

[1] U. Jannsen, M. Rovinsky. Smooth representations and sheaves. *Mosc. Math. J.* 9 (2009), no. 3, to appear. arXiv.math.AG 0707.3914.

[2] U. Jannsen. On finite-dimensional motives and Murre's conjecture. Algebraic Cycles and Motives, (J. Nagel, C. Peters, eds.), *LMS Lect. Notes Series*, 344, vol. 2 (2007), 122–142.

II. Publications submitted for review

[1] U. Jannsen, S. Saito. Bertini and Lefschetz theorems for schemes over discrete valuation rings, with applications to higher class field theory. Submitted 07.11.2009. arXiv.math.AG 0911.1470.

[2] U. Jannsen, S. Saito, K. Sato. Étale duality for constructible sheaves on arithmetic schemes. Submitted 20.10.2009. arXiv.math.AG 0910.3759.

[3] U. Jannsen. Hasse principles for higher-dimensional fields. Submitted 14.10.2009. arXiv.math.AG 0910.2803.

[4] U. Jannsen, S. Saito. Kato conjecture and motivic cohomology over finite fields. Submitted 14.10.2009. arXiv.math.AG 0910.2815.

III. Nonreviewed publications

[1] V. Cossart, U. Jannsen, S. Saito. Canonical embedded and non-embedded resolution of singularities for excellent two-dimensional schemes. Preprint 2009, arXiv:math.AG/0905.2191

A.6.3 PhD Students since 2004

Siamak Firouzian	Zeta functions of local orders (2006)
Moritz Kerz	Milnor K-Theory of semi-local rings (2008)
Felix Schnellinger	Rigid gauges and F-zips, and the fundamental sheaf of gauges G_n (2009)
Thomas Killian	Zeta functions and value semi-groups of orders (on-going)
Marcel Widmann	Displays and gauges over perfect rings (on-going)
Patrick Forré	Class field theory of varieties over local fields with bad reduction (on-going)

A.6.4 Third party funding

MRTN-CT-2003 -504917	Marie-Curie Research training network 'Arithmetic Algebraic Geometry', head of the node Regensburg 02/2004–01/2008
DFG Ja 391/4-2	Étale Kohomologie arithmetischer Schemata (2005–2008)
DFG Ja 391/5-2	P-Torsion in Charakteristik p (2005–2008)
INTAS Ref-Nr. 05-1000008-8118	(2006-2009)
DFG Ja 391/4-3	Endlichkeitssätze in der motivischen Kohomologie (2008–2011)
DFG Ja 391/5-3	Neue Kohomologietheorien in Charakteristik p und 0 (2008–2011)

A.6.5 Other

advisory board of *Mathematische Nachrichten* since 1993
editorial board of *Mathematische Zeitschrift* 2000–2008
member of the senate of the University of Regensburg 10/2004–09/2007 and 2009–2011

A.7 Guido Kings

A.7.1 Curriculum vitae

7.8.1965	born in Cologne
1984-89	Study of Mathematics, University Bonn
1989	Diploma in Mathematics, University Bonn
1989-94	Wissenschaftlicher Mitarbeiter, University Münster
1994	PhD in Mathematics, University Münster
1994-2001	Assistant, University Münster
2000	Habilitation University Münster
2001	Research Professor, MPI für Mathematik Bonn
since 2001	Full Professor University Regensburg
2002	Offer from University Zürich (declined)
2003-05	Dean of NWF-I Mathematik
since 2005	Speaker DFG Research group: Algebraic cycles and L-functions

A.7.2 Publications

I. Reviewed Publications

[1] K. Bannai, G. Kings. p-adic elliptic polylogarithm, p-adic Eisenstein series and Katz measure. To appear in *Amer. J. Math.*, arXiv:0707.3747.

[2] G. Kings. Polylogarithms on curves and abelian schemes. *Math. Z.* 262, No. 3, 527-537 (2009).

[3] G. Kings. Degeneration of polylogarithms and special values of L-functions for totally real fields. *Doc. Math., J. DMV* 13, 131-159 (2008).

[4] J. Hornbostel, G. Kings. On non-commutative twisting in étale and motivic cohomology. *Ann. Inst. Fourier*, 56 (2006), 1257–1279.

II. Publications submitted for review

[1] A. Huber, G. Kings. A cohomological Tamagawa number formula. Submitted August 10th 2009, arXiv:0908.0996.

[2] A. Huber, G. Kings, N. Naumann. Some complements to the Lazard isomorphism. Submitted May 6th 2009, arXiv:0904.3863.

[3] J. Johnson-Leung, G. Kings. On the equivariant and the non-equivariant main conjecture for imaginary quadratic fields. Submitted May 2nd 2008, arXiv:0804.2828.

[4] A. Huber, G. Kings. A p-adic analogue of the Borel regulator and the Bloch-Kato exponential map. Submitted September 3rd 2007, arXiv:math/0612611.

III. Nonreviewed publications

[1] G. Kings. An introduction to the equivariant Tamagawa number conjecture: the relation to the Birch-Swinnerton-Dyer conjecture. Lectures given at the IAS/Park City Math Institute 2009, 31 pages To appear in the Ias/Park City Mathematics Series. http://www.uni-regensburg.de/Fakultaeten/nat_Fak_I/preprints/Preprints2009/26-2009.pdf

A.7.3 PhD Students since 2004

Sandra Eisenreich:	Polylogarithms on modular curves, 2008-on-going
Volker Neumaier:	Motivic description of group cohomology of GL, 2005-on-going
Maximilian Niklas:	Syntomic regulators for modular forms, 2009-on-going
Georg Tamme:	p-adic regulators, 2005-on-going

A.7.4 Third party funding

DFG KI 435/3-1	Sachbeihilfe
DFG KI 435/4-2	FOR 570 Algebraische Zykel und L-Funktionen (Sprecher) Zentralprojekt
DFG KI 435/5-2	36 Monate 1 BAT IIa/halbe
DFG KI 435/6-2	36 Monate 1 BAT IIa und 1 BAT IIa/halbe
DFG KI 435/4-3	FOR 570 Algebraische Zykel und L-Funktionen (Sprecher) Zentralprojekt
DFG KI 435/6-3	24 Monate BAT IIa
DFG KI 435/5-3	36 Monate 2 BAT IIa 75%

A.8 Klaus Künnemann

A.8.1 Curriculum vitae

23.07.1964	born in Lengerich (Westf.)
1992	PhD, Universität Münster (summa cum laude)
1992–93	Post-Doc, IHES, Bures-sur-Yvette/France
1993–97	*Wissenschaftlicher Assistent*, Universität Münster
97	Habilitation, Universität Münster
1997–99	*Hochschuldozent*, Universität Köln
since 1999	Professor, Universität Regensburg
2007-2009	Dean of Naturwissenschaftliche Fakultät I – Mathematik

A.8.2 Publications

I. Reviewed Publications

[1] J.-B. Bost, K. Künnemann. Hermitian vector bundles and extension goups on arithmetic schemes. II. The arithmetic Atiyah extension. In *From probability to geometry.* Volume dedicated to J.M. Bismut for his 60th birthday (X. Ma, editor), Astérisque, to appear; (2008). http://dx.doi.org doi:10.1016/j.aim.2009.09.005

[2] J.-B. Bost, K. Künnemann Hermitian vector bundles and extension goups on arithmetic schemes. I. Geometry of numbers. Advances in Mathematics, to appear; (2007). xxx.uni-augsburg.de/abs/math/0701343

A.8.3 PhD Students

Niels Heinz:	Zulässig metrisierte Geradenbündel und Höhen spezieller Zykel auf Jacobischen, Regensburg 2002
Oliver Meyer:	Über Biextensionen und Höhenpaarungen algebraischer Zykel, Regensburg 2003
Martin Seibold:	Bierweiterungen für algebraische Zykel und Poincarébündel, Regensburg 2007

A.8.4 Awards

Gerhard Hess-Förderpreis, Deutsche Forschungsgemeinschaft, March 1998

A.8.5 Editorial work

Editor *manuscripta mathematica* since March 2006

A.8.6 Third party funding

DFG Ku 818/2	Analoga der Standardvermutungen und Fouriertheorie für arithmetische Chow-Gruppen von abelschen Schemata über Zahlringen (1992-1993)
DFG Ku 818/3	Algebraische Zykel auf arithmetischen Schemata (1998-2002)
DFG Ku 818/4	Arithmetische Erweiterungen und ihre Ext-Gruppen (2005-2008)
DFG Ku 818/5	Arithmetische Schnitttheorie (2005-2008)
DFG Ku 818/5-3	Arithmetische Erweiterungen und Bierweiterungen für algebraische Zykel (2008-2011)

A.9 Niko Naumann

A.9.1 Curriculum vitae

12.11.1973	Born in Marburg/Lahn
1994–99	Student in Munich, Padova and Essen
1999	Diploma in mathematics (advisor: Prof. Dr. G. Frey)
2000–03	PhD student at Münster
2003	PhD in mathematics (advisor: Prof. Dr. C. Deninger)
2004–09	Assistent at the University of Regensburg
2008	Habilitation und Venia Legendi for Mathematics at the University of Regensburg
2008–2009	Bonn (Lehrstuhlverstretung)
1.10.09	Professor at the University of Regensburg

A.9.2 Publications

I. Reviewed Publications

[1] N. Naumann, M. Spitzweck, P. Østvår Motivic Landweber Exactness. Accepted for *Documenta Mathematica.* arXiv:0806.0274

[2] N. Naumann, M. Spitzweck, P. Østvår. Chern classes, K-theory and Landweber exactness over nonregular base schemes, Motives and Algebraic Cycles (a celebration in honour of Spencer Bloch). Fields Institute Communications 56 (2009), 307–319

[3] N. Naumann Arithmetically defined dense subgroups of Morava stabilizer groups Compositio Math. 144, no. 1 (2008), 247–270

[4] F. Baldassarri, C. Deninger, N. Naumann A motivic version of Pellikaan's two variable zeta function Diophantine Geometry Proceedings, Scuola Normale Superiore Pisa, 2007, 35–45

[5] N. Naumann Algebraic independence in the Grothendieck ring of varieties Transactions AMS 359 (2007), no. 4, 1653–1658

[6] N. Naumann The stack of formal groups in stable homotopy theory Advances in Math. 215 (2007), no. 2, 569–600

[7] J. Hornbostel, N. Naumann Beta-elements and divided congruences Amer. J. of Math. 129 (2007), no. 5, 1377–1403

[8] N. Naumann, G. Wiese Multiplicities of Galois representations of weight one Algebra and Number Theory 1, no. 1 (2007) 67–86

[9] N. Naumann Representability of Aut_F and End_F Comm. Algebra 33 (2005), No. 8, 2563–2568

[10] N. Naumann A quantitative sharpening of Moriwaki's arithmetic Bogomolov inequality Math. Res. Lett. 12 (2005), no. 6, 877–884

[11] N. Naumann Linear relations among the values of canonical heights form the existence of non-trivial endomorphisms Canad. Math. Bull. 47 (2004), no. 2, 271–279

II. Publications submitted for review

[1] N. Naumann, M. Spitzweck Brown representability in $\mathbb{A}^1$-homotopy theory. Submitted 09/2009. arXiv:0909.1943

[2] A. Huber, G. Kings, N. Naumann Some complements to the Lazard isomorphism. Submitted 04/2009. arXiv:0904.3863

[3] U. Bunke, N. Naumann The f-invariant and index theory. Submitted 08/2008. arXiv:0808.0257

A.9.3 PhD Students since 2003

- none -

A.9.4 Third party funding

- Heisenbergstipendium (March 2009)

A.10 Alexander Schmidt

A.10.1 Curriculum vitae

5.12.1965	born in Berlin
1984	Abitur, Heinrich-Hertz-Oberschule Berlin
1991	Diplom in Mathematics, Humboldt-University Berlin
1993	PhD Thesis, Heidelberg University
2000	Habilitation at Heidelberg University
since 2004	Professor of Mathematics, University of Regensburg

A.10.2 Publications

Ia. Reviewed Publications in scientific journals

[1] M. Kerz, A. Schmidt. On different notions of tameness in arithmetic geometry. *Math. Ann.*, to appear. arXiv:0807.0979

[2] A. Schmidt. Über Pro-p-Fundamentalgruppen markierter arithmetischer Kurven *J. reine u. angew. Math.*, to appear. arXiv:0806.0772

[3] M. Kerz, A. Schmidt. Covering data and higher dimensional class field theory. *J. of Number Theory*, 129 (2009), 2569–2599.

[4] A. Schmidt. On the $K(\pi, 1)$-property for rings of integers in the mixed case. *RIMS Kokyuroku Bessatsu*, B12 (2009), 91–100.

[5] J.A. Hillman, A. Schmidt. Pro-p groups of positive deficiency. *Bull. London Math. Soc.*, 40 (2008), 1065–1069.

[6] A. Schmidt. Rings of integers of type $K(\pi, 1)$. *Doc. Math.*, 12 (2007), 441–471.

[7] A. Schmidt. Singular homology of arithmetic schemes. *Algebra & Number Theory*, 1 (2007), 183–222.

[8] A. Schmidt. Some Consequences of Wiesend's Higher Dimensional Class Field Theory. *Math. Zeitschrift*, 256 (2007), 731–736.

[9] A. Schmidt. Circular sets of prime numbers and p-extensions of the rationals. *J. reine und angew. Math.*, 596 (2006), 115–130.

[10] A. Schmidt. Higher dimensional class field theory from a topological point of view. *Algebraic number theory and related topics (RIMS Proceedings)*, 10 (2005), 168–178.

[11] A. Schmidt. Tame class field theory for arithmetic schemes. *Invent. Math.*, 160 (2005), 527–565.

Ib. Monographs

[1] J. Neukirch, A. Schmidt, K. Wingberg. Cohomology of Number Fields. 2nd ed. *Grundlehren der Mathematischen Wissenschaften* 323, Springer Verlag Berlin, Heidelberg, New York (2008).

[2] A. Schmidt. Einführung in die algebraische Zahlentheorie. Springer-Lehrbuch, Springer Verlag Berlin Heidelberg (2007).

III. Nonreviewed publications

[1] A. Schmidt, K. Wingberg. Extensions of profinite duality groups. Preprint (2008). arXiv:0809.4371

[2] A. Schmidt. On the étale homotopy type of Morel-Voevodsky spaces. Preprint (2005). http://www.mathematik.uni-r.de/Schmidt/papers/schmidt19-de.html

A.10.3 Ph. D. Students since 2004

F. Klössinger:	Ramification groups in higher class field theory, 2009-ongoing
M. Sigl:	L^2-Betti numbers of arithmetic fundamental groups, 2009-ongoing
J. to Baben:	Fibre sequences in étale homotopy, 2007-ongoing
S. Wolfrath:	Die scheingeometrische étale Fundamentalgruppe, Regensburg 2009

A.10.4 Third party funding

DFG Schm 1134-4	Heisenbergstipendium (2002-2004)
DFG Schm 1134-5	Zahme Fundamentalgruppen (2002-2005)
DFG Schm 1134-6	Höherdimensionale Klassenkörpertheorie (2005–2008)
DFG Schm 1134-7	$\mathbf{A}^1$-Homotopie und Arithmetik (2005–2008)
DFG Schm 1134-8	Homotopieinvarianten arithmetischer Schemata (2008–2011)
Isaac Newton Institute	Visiting Fellowship, Fall 2009