Gursky, Matthew J.

Obstructions to the existence of conformally compact Einstein manifolds

In this talk I will describe a singular boundary value problem for Einstein metrics. This problem arises in the Fefferman-Graham theory of conformal invariants, and in the AdS/CFT correspondence. After giving a brief overview of some important results and examples, I will introduce an index-theoretic invariant which gives an obstruction to existence in the case of spin manifolds. This is joint work with Q. Han and S. Stolz. If time permits I will also mention recent work with G. Szekelyhidi on a "local" version of the existence question.

Matsumoto, Yoshihiko

A construction of Poincaré-Einstein metrics of cohomogeneity one on the ball

Poincaré-Einstein metrics and their relationship to conformal structures on the boundary at infinity have attracted much interest since 1980s, but the known results on the existence of such metrics are still quite limited. Apart from the Graham-Lee theorem and its relatively slight generalizations, concerning deformations of the hyperbolic metric and some others, what are known are several constructions in individual cases. After sketching the current state of the matter, I will discuss yet another explicit construction of a one-parameter family of Poincaré-Einstein metrics, which is based on a method of Page and Pope, hoping that it sheds some light to the space of all Poincaré-Einstein metrics. The construction has the charm that it takes place on the unit ball of arbitrary even dimension, which is not shared by other constructions, and the family contains the hyperbolic metric itself. Somewhat miraculously, as the limit of the family the complex hyperbolic metric emerges.

Futaki, Akito

Conformally Kähler, Einstein-Maxwell metrics

Let M be a compact complex manifold admitting a Kähler structure. A conformally Kähler, Einstein-Maxwell metric (cKEM metric for short) is a Hermitian metric $\tilde{g}$ on M with constant scalar curvature such that there is a positive smooth function f with $g = f^2\tilde{g}$ being a Kähler metric and f being a Killing Hamiltonian potential with respect to g. Fixing a Kähler class, we characterize such Killing vector fields whose Hamiltonian function f with respect to some Kähler metric g in the fixed Kähler class gives a cKEM metric $\tilde{g} = f^{-2}g$. The characterization is described in terms of critical points of certain volume functional. The conceptual idea is similar to the cases of Kähler-Ricci solitons and Sasaki-Einstein metrics in that the derivative of the volume functional gives rise to a natural obstruction to the existence of cKEM metrics. However, unlike the Kähler-Ricci soliton case and Sasaki-Einstein case, the functional is neither convex nor proper in general, and often has more than one critical points.

Botvinnik, Boris

Positive scalar curvature on manifolds with fibered singularities

This is joint work with J. Rosenberg. A compact manifold with fibered P-singularities is a possibly singular pseudomanifold M_Σ with two strata: an open nonsingular stratum $\mathring{M}$ a smooth open manifold and a closed stratum βM, such that a tubular neighborhood of βM is a fiber bundle with fibers each looking like the cone on a fixed closed manifold P. We discuss what it means for such an M_Σ with fibered P-singularities to admit an appropriate Riemannian metric of positive scalar curvature, and we give necessary and sufficient conditions the necessary conditions based on suitable versions of index theory, the sufficient conditions based on surgery methods and homotopy theory for this to happen when the singularity type P is either Z/k or S^1, and M and the boundary of the tubular neighborhood of the singular stratum are simply connected and carry spin structures. Along the way, we prove some results of perhaps independent interest, concerning metrics on spin^c manifolds with positive "twisted scalar curvature," where the twisting comes from the curvature of the spin^c line bundle.

Carron, Gilles

A Bonnet-Myers theorem under a spectral condition

This is joint work with C. Rose (TU Chemnitz, Germany). According to the Bonnet-Myers theorem, a complete Riemmannian manifold with Ricci curvature bounded from below by a positive constant is closed and has finite fundamental group. We obtain that a closed Riemannian manifold for which a certain Schrodinger operator is positive has finite fundamental group.

Kröncke, Klaus

The moduli space of Ricci-flat metrics

We say that a Ricci-flat metric on M is structured if its pullback to the universal cover admits a parallel spinor. The holonomy of these metrics is special as these manifolds carry some additional structure, e.g. a Calabi-Yau structure or a G_2-structure. We will show that the premoduli space of compact structured Ricci-flat metrics is a smooth manifold. Moreover, the holonomy group and the dimension of the space of prallel spinors is constant along connected components.

This result builds up on previous work by Nordström, Goto, Koiso, Tian & Todorov, Joyce and Wang. The important step is to pass from irreducible to reducible holonomy groups. We will also discuss applications of this result for Ricci flow, for the topology of the space of nonnegative and positive scalar curvature metrics and in Lorentzian geometry. This talk is based on joint work with Bernd Ammann, Olaf Müller, Hartmut Weiß and Frederik Witt.

Humbert, Emmanuel

Mass functions on a manifold

I will present some recent results we obtained with Andreas Hermann. Let M be a compact manifold of dimension n larger than 3. We introduce the mass functions as follows : for $a > 0$, it is computed as the supremum/infimum of the masses at some point p of M (i.e. the constant term at p of the Green function for the Yamabe operator) over the set of points p and metrics g which are flat on the ball $B^g(p, 1)$ and whose Yamabe constant is greater than a. We show several interesting properties of these functions.

Lohkamp, Joachim

The Splitting Theorem in Scalar Curvature Geometry

We derive the long conjectured **splitting theorem**: Let (M^n, g), $n \geq 3$, be a smooth, compact Riemannian manifold with $scal(g) > 0$ and $\alpha \in H_{n-1}(M; \mathbb{Z})$ be given. Then, there is a **smooth** compact hypersurface $H^{n-1} \subset M^n$ representing α, i.e. $[H] = \alpha$, so that H admits a smooth $scal > 0$-metric. This implies the fact that enlargeable manifolds cannot admit $scal > 0$-metrics. Also it readily implies both, the Riemannian and the space-time, positive mass theorems. The results are based on the theory of hyperbolic unfoldings of minimal hypersurfaces.

Herzlich, Marc

Universal positive mass theorems

We revisit Witten's proof of the positive mass theorem in a very general setting where the spinor bundle and the Dirac operator are replaced by any choice of an irreducible natural bundle and a very large choice of first-order operators. As an application, we show that it may lead to a positive mass theorem along the same lines if the necessary curvature conditions are satisfied.

Dahl, Mattias

Mass-like invariants for asymptotically hyperbolic metrics

Asymptotically Euclidean and asymptotically hyperbolic manifolds have mass invariants computed at infinity. These invariants have the interpretation as the total mass of the manifold as a slice of spacetime in general relativity. One can ask if there are other geometric invariants at infinity of such manifolds.

In this talk we will consider the asymptotically hyperbolic case and we will formulate a definition of mass-like invariants at infinity for such metrics. Further, we will classify the set of mass-like invariants for asymptotically hyperbolic metrics. It turns out that the standard mass is one example among two families of invariants.

This is joint work in progress with Julien Cortier and Romain Gicquaud.

de Lima, Levi Lopes

The mass of asymptotically hyperbolic manifolds with a noncompact boundary

We discuss a positive mass inequality (and its consequences) for the class of manifolds in the title, under the spin assumption. This is a natural extension to this setting of a previous result by P. Chrusciel and M. Herzlich, who treated the boundaryless case. Joint work with S. Almaraz.

Kazaras, Demetre

Minimal hypersurfaces with free boundary and PSC bordism

Let (Y, g) be a compact Riemannian manifold of positive scalar curvature (psc). It is well known, due to Schoen-Yau, that any closed stable minimal 2-sided hypersurface of Y also admits a psc-metric. We establish an analogous result for stable minimal hypersurfaces with free boundary. Furthermore, we combine this result with tools from geometric measure theory and conformal geometry to study psc-bordism. For instance, assume (Y_0, g_0) and (Y_1, g_1) are closed psc-manifolds equipped with stable minimal hypersurfaces X_0 and X_1. Under natural topological conditions, we show that a psc-bordism between (Y_0, g_0) and (Y_1, g_1) gives rise to a psc-bordism between X_0 and X_1 equipped with the psc-metrics given by the Schoen-Yau construction.

Tadano, Homare

tba

Xu, Tian

Spinor field equations and the problem of prescribing mean curvature on S^2

In this talk, we shall consider the existence of solutions for the equation $D\psi = Q(x)|\psi|^{\frac{2}{m-1}}\psi$ on (S^m, g_{S^m}), $m \geq 2$, where Q is a C^2 positive function. We prove that the set of Q's for which a solution exists is dense, in C^1-topology, in the space of positive bounded smooth functions. When $m = 2$, we relate the zero sets of a solution with the genus of a Riemannian surface. As a consequence, a prescribed mean curvature embedding theorem of $S^2 \to \mathbb{R}^3$ is established.

Petean, Jimmy

Low energy nodal solutions of the Yamabe equation on the sphere

Positive solutions of the Yamabe equation on the sphere are well known. They are all equivalent and minimize the energy. The family of sign-changing solutions is much richer. The energy of such solutions can be arbitrarily large, and it is not known what is the minimum of their energies. In the talk I will discuss joint work with Juan Carlos Fernandez on the construction of nodal solutions. Given any isoparametric hypersurface M on the sphere and a positive integer k we prove the existence of a solution whose nodal set is given by k copies of M. The solutions are obtained by studying the double shooting problem for the corresponding ODE, and their energies can be computed numerically.

Ruiz, Juan Miguel

Multipeak solutions for the Yamabe equation

Let (M, g) be a closed Riemannian manifold of dimension $n \geq 3$, $x_0 \in M$ an isolated local minimum of the scalar curvature s_g of g, and k any positive integer. We will talk about positive solutions with k-peaks, which concentrate around x_0, of the subcritical Yamabe equation

$$-\epsilon^2 \Delta_g u + (1 + c_N\, \epsilon^2 s_g) u = u^q,$$

where $c_N = \frac{N-2}{4(N-1)}$ for an integer $N > n$, $q = \frac{N+2}{N-2}$ and $\epsilon > 0$ is small enough.

This provides solutions to the Yamabe equation on Riemannian products $(M \times X, g + \epsilon^2 h)$, where (X, h) is a Riemannian manifold of dimension $(N - n)$, with constant positive scalar curvature. This is joint work with Carolina Rey.

Mondello, Ilaria

Non-existence of Yamabe minimizers in a singular setting

In the recent years there have been significant progress in the study of the Yamabe problem, that is looking for a constant scalar curvature metric in a conformal class, in a singular setting. An example of non-existence was given by J.Viaclovsky in the case of one isolated orbifold singularity, while K.Akutagawa, G.Carron and Rafe Mazzeo showed existence when assuming a strict generalized Aubin's inequality. After giving a brief introduction about the singular spaces we deal with and the main issues arising when trying to solve the Yamabe problem in this setting, I will present a non-existence result for Yamabe minimizers on a manifold with non-isolated conical singularities. This is based on a joint work with K.Akutagawa.

Bettiol, Renato G.

Non-uniqueness of conformal metrics with constant Q-curvature

The problem of finding (complete) metrics with constant Q-curvature in a prescribed conformal class is an important fourth-order cousin of the Yamabe problem. In this talk, I will provide some background on Q-curvature and discuss how several non-uniqueness results for the Yamabe problem can be transplanted to this context. However, special emphasis will be given to multiplicity phenomena for constant Q-curvature that have no analogues for the Yamabe problem, confirming expectations raised by the lack of a maximum principle. This is based on joint work with P. Piccione and Y. Sire.

Große, Nadine

On the first eigenvalue on degenerating hyperbolic surfaces

We consider the first non-zero eigenvalue λ_1 of the Laplacian on hyperbolic surfaces for which one disconnecting collar degenerates and prove that $8\pi\nabla\log\lambda_1$ essentially agrees with the dual of the differential of the degenerating Fenchel-Nielson length coordinate. This result is mainly based on analysing properties of holomorphic quadratic differentials and relating quadratic holomorphic differentials to Fenchel-Nielson coordinates. As a corollary we obtain that λ_1 essentially only depends on the length of the collapsing geodesic σ and the topology of $M \setminus \sigma$, with error estimates that are sharp for all surfaces of genus greater than 2. Our result improves corresponding bounds obtained by Burger. This is joint work with Melanie Rupflin (Oxford), arXiv:1701.08491.