MINI-COURSE ON STABLE HOMOTOPY THEORY
(1-5 APRIL 2019)

Program

MONDAY, APRIL 1st

Talk 1 (13.30-14.30): *Introduction to spectra* (KEVIN): The purpose of this talk is to introduce the category of spectra. We will follow the classical approach using CW spectra. Introduce the basic definitions and give many examples. This includes: (CW-)spectra, Ω-spectra, maps between spectra, homotopies, homotopy groups of spectra, and (weak) homotopy equivalences. Main references are [1, III.2] and [2, Ch. 8].

Coffee break! (14.30-15)

Talk 2 (15-16): *Homotopy theory of spectra* (BENEDIKT): The purpose of this talk is to discuss some important properties of the homotopy category of spectra. This includes the property that it is additive and that cofiber and fiber sequences coincide. Main references are [1, III.3], [2, Ch. 8] (and [3] for further reading).

TUESDAY, APRIL 2nd:

Talk 3 (13.30-14.30). *Spectra and generalized (co)homology* (DANIEL): The purpose of this talk is to define the generalized homology and cohomology theories associated with a spectrum E and discuss some examples including ordinary singular (co)homology and bordism theories. The example of E-(co)homology with coefficients should also be discussed. Main references are [1, III.6], [2, Ch. 8 and 9] and [4, 3.4].

Coffee break! (14.30-15)

Talk 4 (15-16). *Brown representability* (JULIAN): This talk is concerned with the Brown representatibility theorem. State and prove the (unstable) Brown representability theorem. Deduce the stable version and explain how this establishes an equivalence between spectra and generalized cohomology theories. If time permits, the representability theorem of Adams could also be mentioned (without proof). Main reference is [2, Ch. 9].

WEDNESDAY, APRIL 3rd

Talk 5 (13.30-14.30). *Spanier-Whitehead duality* (JONATHAN): The purpose of this talk is to explain the Spanier-Whitehead duality for finite spectra. Any required properties of the smash product of spectra will be stated without proof. Define the duality and deduce some its main properties. Then discuss the case of duality for a closed smooth manifold. If time permits, the representability of generalized homology theories could be sketched. Main references are [1, III.5], [2, pp. 321-335] and [5, Ch. 7].

Coffee break! (14.30-15)

Talk 6 (15-16). *Smash product* (MARKUS).

THURSDAY, APRIL 4th

Exercise session (11-12.30). *Review of model categories*: In this class we review the definition of a model category and we define the Quillen functor. We describe some basic properties (witout proofs), for example the existence of derived functors and the homotopy category. Define also pointed and stable model categories. Main reference is [6].

Talk 7 (13.30-14.30). *Model category of Bousfield–Friedlander and of symmetric spectra* (SPEAKER): References are [7], [8] and [9].

Coffee break! (14.30-15)

Talk 8 (15-16). *Infinite loop spaces,* Γ*-spaces, etc.* (GEORGE).

FRIDAY, APRIL 5th

Talk 9 (13.30–). *Introduction to stable* ∞*-categories* (KIM).

References

[1] Adams, J. F., *Stable homotopy and generalised homology.* Reprint of the 1974 original, Chicago Lectures in Mathematics, University of Chicago Press, Chicago, IL, 1995.
[2] Switzer, R. M., *Algebraic topology - homotopy and homology.* Reprint of the 1975 original. Classics in Mathematics. Springer-Verlag, Berlin, 2002.
[3] Margolis, H. R., *Spectra and the Steenrod algebra. Modules over the Steenrod algebra and the stable homotopy category.* North-Holland Mathematical Library, No. 29. North-Holland Publishing Co., Amsterdam, 1983.
[4] Kochman, S. O., *Bordism, stable homotopy and Adams spectral sequences.* Fields Institute Monographs Vol. 7. American Mathematical Society, Providence, RI, 1996.
[5] tom Dieck, T., *Algebraic topology. EMS Textbooks in Mathematics.* European Mathematical Society (EMS), Zu rich, 2008.
[6] Hovey, M., *Model Categories.* Mathematical Surveys and Monographs, vol. 63, American Mathematical Society, Providence, RI, 1999.
[7] Bousfield, A., Friedlander, E., *Homotopy theory of* Γ*-spaces, spectra, and bisimplicial sets*, Springer Lecture Notes in Math., Vol. 658, Springer, Berlin, 1978, pp. 80-130.
[8] Schwede, S., *Spectra in model categories and applications to the algebraic cotangent complex*, Journal of Pure and Applied Algebra 120 (1997) 77-104.
[9] Mandell, M., May, P., Schwede, S., Shipley, B., *Model categories of diagram spectra*, Proceedings of the London Mathematical Society, 82 (2001), 441-512.

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